Using Hicks Model to Verify the Stability and Equilibrium into Stock Market System

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The variations of the system – as a result of the stock market reactions to the outside stimulants and the internal parametric modifications – that results in a modification of the system's state, determines the system's behavior.
This is the motive for authors propose Hicks model to study stability and equilibrium into bourses system.
Keywords*: stability, equilibrium, system, dynamic, Hicks model.*

The stability and equilibrium into the stock market system

The structure of the system and his connections determine a certain evolution of it and measured at a certain moment (the value of the parameters, which are instruments of regulation of the system), represents the system's state.

If it's admitted that any behavior is a form of manifestation of the system – environment interaction, than the equilibrium state of the system is the expression of compatibility of the decisions taken by the management system, following a relative compatibility between the two, which can be maintained for a certain period of time.

Looking at it from the manifestation point of view we can observe the *static and* dynamic equilibrium. Considered as an work apotheosis, so with no effective correspondent in reality, the static equilibrium can be characterized thru the manifestation of almost unseen changes, insignificant between the different processes or structures of the stock market, in such a way that the general state of it remains unchanged..

The necessary condition for the stock market system to be in a state of static equilibrium is: $Y = G$, which means that the offer of financial assets is equal to the offer with the demand of assets, in such a way that there is no offer excess ($\Delta Y = 0$) or global demand ($\Delta G = 0$).

The dynamic equilibrium of the stock market system is defined by the modification of it's state under the contradictory action of the relation between the demand and offer and resources and demand, manifesting over a short or long period of time.

The dynamic of the stock system implies the fact that time represents an essential variable which cannot be slowed down or inversed. The sources of the dynamics of the stock market system are found in perturbations manifested on the market by various factors endogenous or exogenous, in the decision policies practiced by the management system. The condition of existence of the dynamic equilibrium: $Y \neq G$, excesses (ΔY and ΔG), or the deficiencies of demand or offer ($-\Delta Y$ and $-\Delta G$) are solved, in time, by proper use of the market interventions with the sole purpose of decreasing the excesses or deficiencies.

The stability and equilibrium of the stock market system constitutes a permanent preoccupation of the management system. The stock market manager is interested in obtaining certain stability and a real equilibrium of the market which he organizes in the purpose of making transactions with financial assets.

Starting from this idea, in this paragraph we want to model the activity of the stock market system with the help of certain general models of stability and economic equilibrium [1].

Model for the study of the stability and equilibrium of the stock market system

The general models of stability and equilibrium, of the stock market system are versions of classical models of regulation of the economic processes. From all of them we have stopped at the model proposed by Hicks.

▪ **The Hicks model** is a complex model, it is an accelerator type of dynamical equilibrium to of the national product, based on the following relations:

Y(t) = C(t) + I(t) + G(t)
C(t) = a Y(t-1)
I(t) = b (Y(t-1) – Y(t-2))

At the level of system object – Market of financial derivates – the parameters of the Hicks model modify the variables of the system are:

Y(t) = offer assets	a = multiplier
C(t) = values transactions	b= accelerator
I(t) = investments	
G(t) = demand assets	

Starting from the Hicks [5], we exemplify a model of economic dynamics described by the ordinary differential equation (II order):

$\dot{Y}+(1+s-v)\dot{Y}+sY=G$, which can be put under the form of an dynamic system with the help of the relation $Y=\dot{X}$.

Using the relation above, we obtain the following system of two equations, representing the processes that take place in the same time, (t) defining a structure of relations between the variables Y and X of the stock market system.

$$\begin{cases} X = Y \\ Y = -sX - (1+s-v)Y + G \end{cases}$$

where
Y = the offer of financial assets;
G = the demand for financial assets;

In the case of the stock market system, as in any real system the simple comparison of the two system variables (Y and X) is difficult, being perturbed by the effects other variables which occur within the system. Because of this, in this model:

- ***1/s*** is called multiplier – tendency towards transitioning;
- **v** accelerator.

It is imposed to exemplify the necessity of introducing the two parameters (1/s and v) because these will influence the general behavior of the system. The correlation which is formed in the stock market time and space between demand, offer, investments, profit, and transactions is under the direct influence of the principals of the multiplier and accelerator. We have identified the multiplier 1/s as being tendency towards investment starting from the principals of the multiplier [5], according to the economic theory and used frequent in the activity of modeling a system. Thru it's meaning, the multiplier, proves the direct connection between the entries in the stock market – investments– and it's exists, under the form of income for the participants at the stock market activity..

The principal of the multiplier expresses the interaction which is formed between the income growth (ΔY) and the growth of stock market investment (ΔI), under the form of a coefficient of amplification which shows the size of the growth in income, obtained thru transactions at the growth with one unit of the sums invested. The stock market investment compel the transactions in the stock market ring, generating incomes – but attention – only in the conditions of stability and equilibrium.

The principal of the accelerator expresses the effect that income growth has over investment in the sense that the growth with one unit of the demand of financial assets (ΔG) begins to force a direct growth of the offer. Between the demand of assets and investments there is a acceleration relationship. Calculated as a fraction between the size of investment during one year I(t) and the variation of the income during the previous period, the accelerator tries to give a theoretic explication of the variation in income growth.

We will study mathematically the behavior in time of this system remembering that economically looking at it: $s \in (0,1)$ and $v > 0$.

The dynamic of the economical system supposes determining the points of equilibrium and the zones of stability inside the space of the parameters (s, v, G).

Points of equilibrium

The points of equilibrium are the solutions of the algebraic system:

$$\begin{cases} \dot{X} = 0 \\ \dot{Y} = 0 \end{cases} \Rightarrow \begin{cases} Y = 0 \\ Y = 0 \end{cases} \Rightarrow \begin{cases} \mathrm{sX - (1 + s - v)Y + G = 0} \\ X = \dfrac{G}{s} \end{cases}$$

So, for s ≠ 0, G ≠ 0 the system admits only one point of equilibrium ($\frac{G}{s}$, 0). for s = 0, G = 0 admits an infinity of equilibrium points and if s= 0, G ≠ 0 the system does not admit equilibrium points.

The following picture is used to visualize the Σ stability surface given by the equation:

$\mathbf{s^2 + v^2 - 2sv - 2s - 2v + 1 = 0}$ *in the parameter space* (v, s, G)

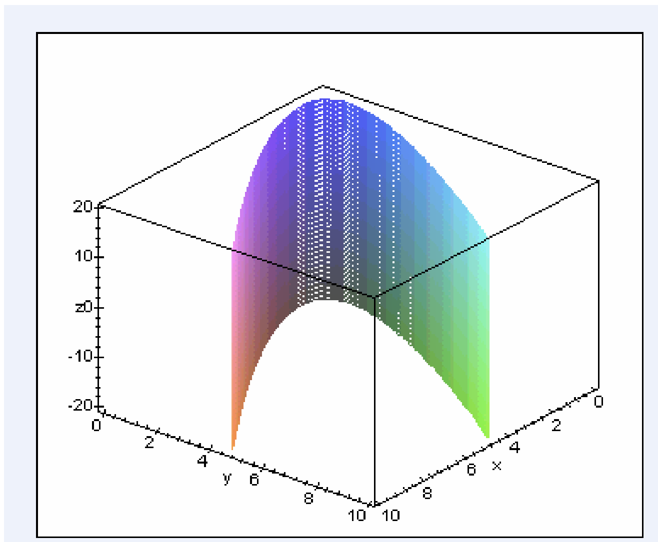


Fig.1. Σ Surface in the parameter space (v, s, G)

As seen, this surface delimitates the stability zones and the instability zones.

We will study the system behavior around the equilibrium u_0 = ($\frac{G}{s}$, 0). The initial system is an affine system and his linearity around the equilibrium point is:

$$\begin{cases} \dot{X} = Y \\ \dot{Y} = -sX - (1 + s - v)Y \end{cases}$$

The attached matrix will be $A = \begin{pmatrix} 0 & 1 \\ -s & -(1 + s - v) \end{pmatrix}$, and the characteristic equation is:

$\lambda^2 + (1+s-v)\lambda + s = 0$, with discriminant Δ.

We will discuss the sign of Δ, considering Δ = 0, equation in s:

$\Delta' = 4(1+2v+v^2) - 4(v^2-2v+1) = 16v$

If v>0 => Δ'>0 and the equation Δ = 0 has the following solutions:

$$s_2 = \frac{2(1+v)+4v}{2} = 1 + v + 2\sqrt{v} = \left(1+\sqrt{v}\right)^2 > 1$$

$$s_1 = \frac{2(1+v)-4\sqrt{v}}{2} = 1 + v - 2\sqrt{v} = \left(1-\sqrt{v}\right)^2$$

The two parameters **s** and **v** determine the stability of Y_t, according to the nature and signs of λ_1 and λ_2, therefore in the three cases in which the determinant Δ is positive, null or negative

Table 1

S	The solutions of the characteristic equation	Type of equilibrium point	Region
$0<s<s_1$	$\Delta>0,\ \lambda_{1,2}=\dfrac{1+s-v\pm\sqrt{\Delta}}{2}\in R$	P = s>0: 1+s-v<0 node repulse	A
		P = s>0: 1+s-v>0 node attractive	
$s=s_1$	$\Delta=0,\ \lambda_1=\lambda_2=1-\sqrt{v}$	v>1 node attractive	1
		v<1 node repulse	
$s_1<s<s_2$	$\Delta<0,\ \lambda_{1,2}=\dfrac{-(1+s-v)\pm i\sqrt{\Delta}}{2}$	1+s-v<0 focal point repulse	B
		1+s-v=0 solution periodic	2
		1+s-v>0 focal point attractive	C
$s=s_2$	$\Delta=0,\ \lambda_1=\lambda_2=1+\sqrt{v}>0$	node attractive with only one direction asymptotic	3
$s>s_2$	$\Delta>0,\ \lambda_{1,2}=\dfrac{1+s-v\pm\sqrt{\Delta}}{2}\in R$	P = s >0 node repulse	D

Conclusions

The solutions offered by the model have been synthesized with the help of Table 1. These results allow the determination of the type of movement around the equilibrium point, underlining the complex situations that may occur. For an easier interpretation we will consider *Figure 2* on which we have identified the four regions:

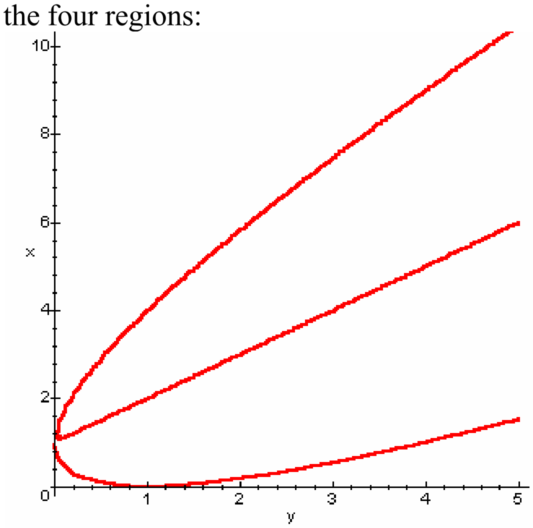
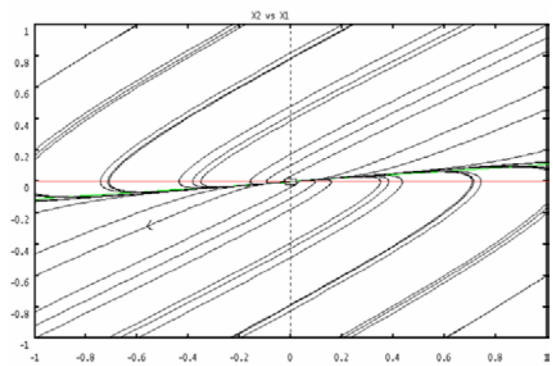


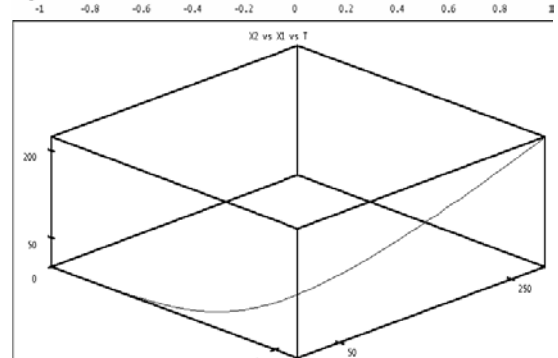
Fig.2. Delimitation of the regions in the plane (v, s)

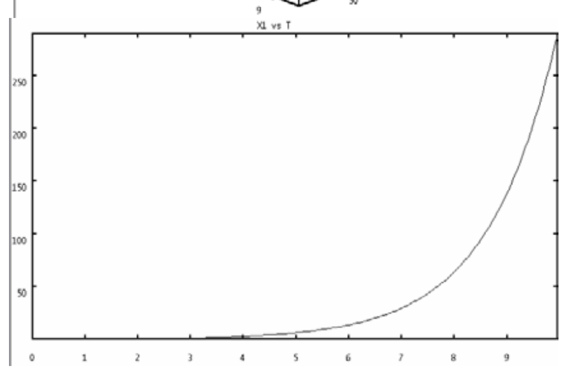
Region A – the stability condition is not satisfied because:

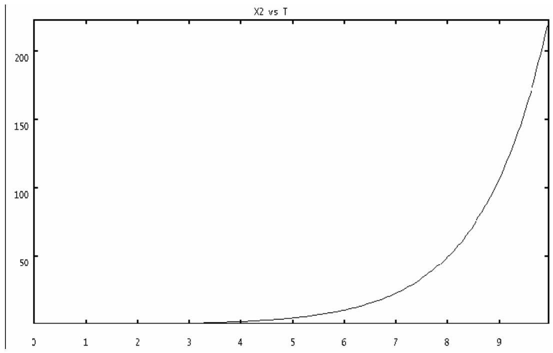
$$1+s-v>0 \text{ and } s>(1+\sqrt{v})^2.$$

The movement is momentarily explosive.

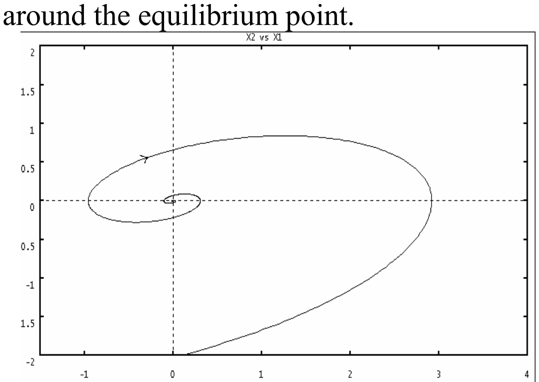


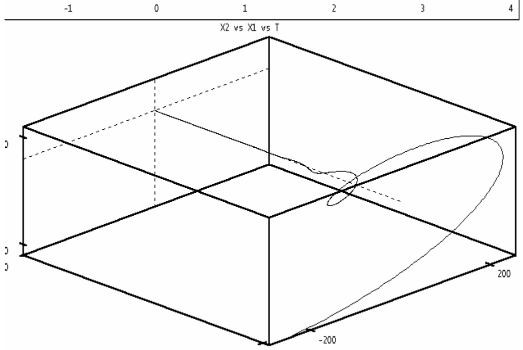


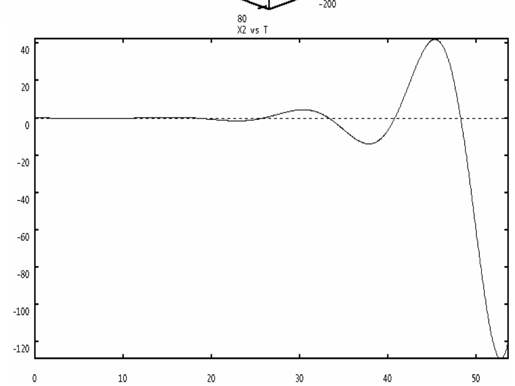


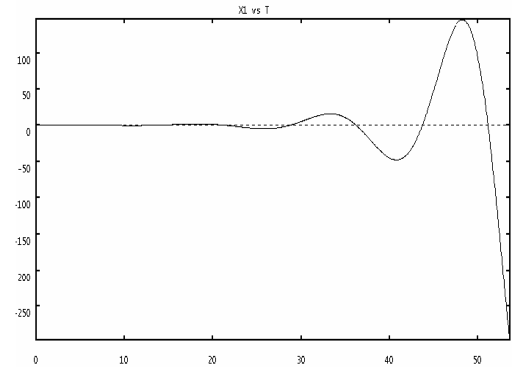


Region B – In this case $1+s-v>0$, $(1-\sqrt{v})^2 < s > (1+\sqrt{v})^2$, the stability condition being satisfied, the roots λ_1, λ_2 are complex with the real part not null but positive. An explosive oscillation will take place around the equilibrium point.





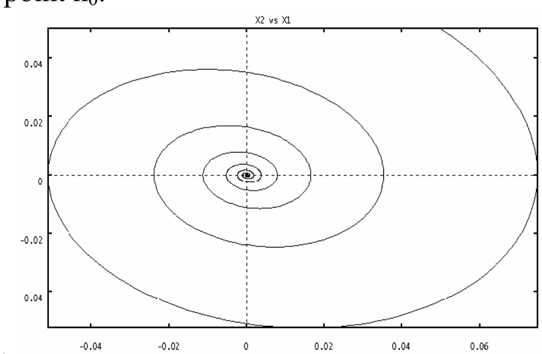


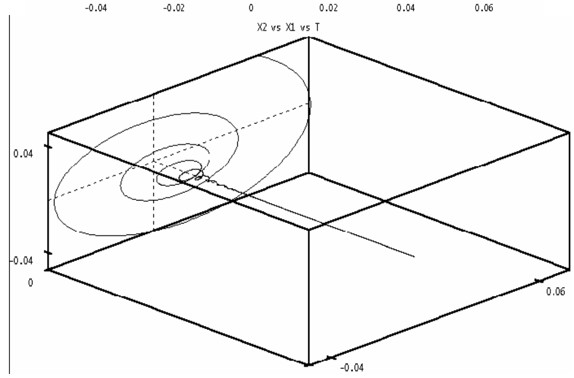


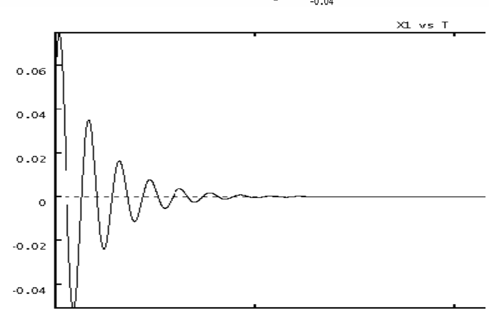
Region C – has $1+s-v<0$ and

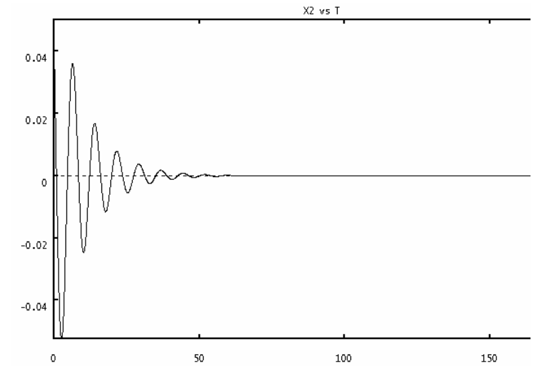
$$(1-\sqrt{v})^2 < s > (1+\sqrt{v})^2,$$

the roots being complex, with the real part not null, negative. Therefore the stability condition is satisfied, obtaining an amortization oscillation around the equilibrium point n_0.

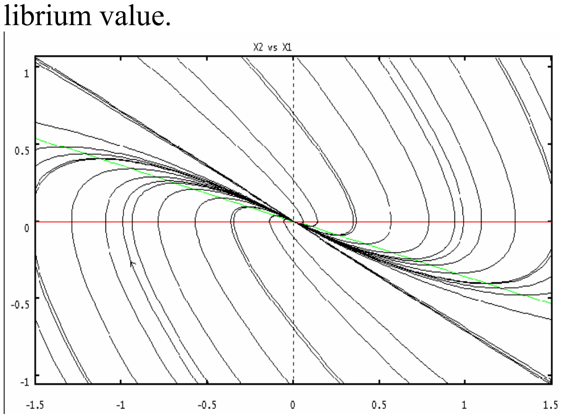


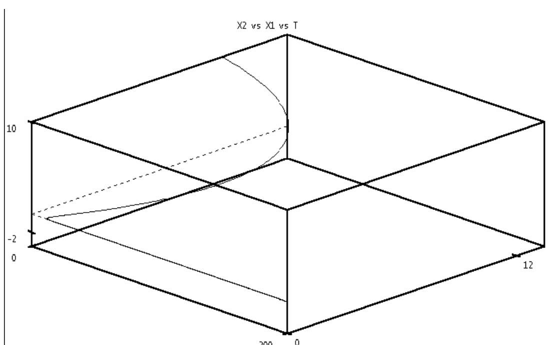


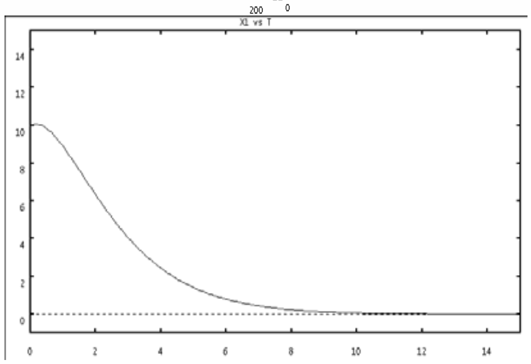


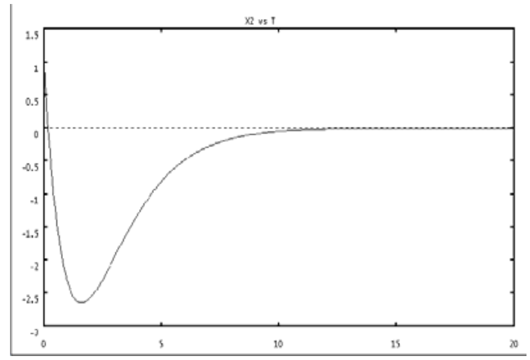


Region D – is determined by the following conditions $1+s-v<0$ and $0<s<(1-\sqrt{v})^2$. The stability condition is satisfied, the roots λ_1, λ_2 are real and negative. The stock market system will have a monotone motion converge towards the equilibrium value.



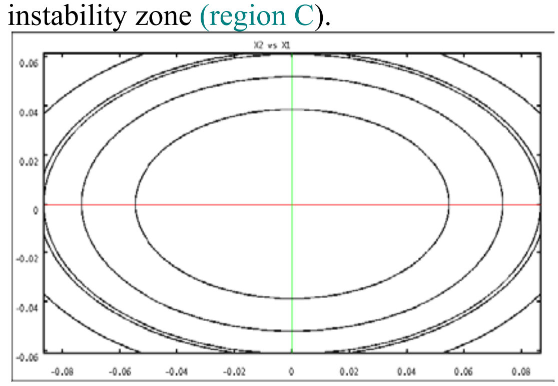


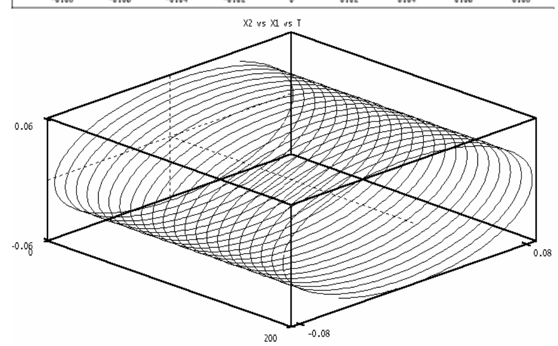


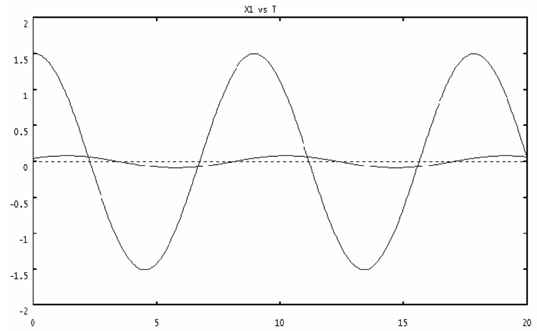


Particular case 1 – in this situation, the movement is dominated by a monotone movement infinite.

Particular case 2 – For the points situated on the separation line between region **B** and region **C**, $\lambda_1, \lambda_2 \in C$ with Realλ_1 = Realλ_2 = 0 therefore the characteristic roots are purely imaginary, which a constant magnitude of the oscillation. In this case the movement is neutrally stabile and the delimitation line separates the stability zone **(region B)** from the instability zone (region C).







Particular case 3 – For the points on the separation line between **region A** and **region B** we will have $\lambda_1 = \lambda_2 = \lambda < 0$ and the movement converge monotone towards equilibrium.

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