CAMUCRA - A Polynomial Clearing Algorithm for Multi-Unit Combinatorial Reverse Auction

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In this paper, we debate the resource allocation problem of the multi-unit combinatorial auctions and we use a solution based on polynomial algorithms for clearing rules. These types of algorithms are called polynomial because they have polynomial complexity and they seem to be optimum.
Our approach of the polynomial clearing algorithms is related to reverse auctions only, in which there is just one buyer and multiple sellers. This algorithm determines the winner for a general case, thus having a large application and allowing the exploitation of the combinatorial auctions advantages in real-life online auctions.
Although the algorithm does not guarantee to find the best allocation solution, it provides a method for the determination of an acceptable, profitable solution closed to the optimum solution for the resource allocation problem of the multi-unit combinatorial auctions.
***Keywords**: electronic reverse auction, polynomial algorithms, clearing rules, multi-unit combinatorial auctions*

1 Introduction

Auctions have long been a popular method for buying and selling products and services. With the advent of the Internet and the proliferation of Web users, auctions moved online and gained in popularity because they reduce transaction costs for both suppliers and buyers.
While forward auctions feature increasing incremental bidding, reverse auctions feature decreasing incremental bidding [Parente2001].
Electronic reverse auction is a dynamic, real-time negotiation between a purchasing organization and several suppliers competing online against one another to win the opportunity to supply goods and/or services to the purchasing organization [Kirkless2003]. At the end of the event, the lowest bid wins the auction.
Reverse auctions are used in industries where companies often send requests for proposals or requests for quotes to gather bids for business contracts or supplies.
In order to choose a reverse auction, we present some of the different types of reverse auction approaches and pricing models. There are two basic types of pricing models: a buyer-paid model and a seller-paid model. In the first model, the buyer controls the process and the decision as to which suppliers are qualified to participate and bid. In the second model, the buyers can post their requirements and sellers who are paid members of the system can view the requirements and place bids on them [eDyna].
In a typical reverse auction, sellers bid to provide a product/service nominated, requested by the buyer. The buyer describes to an auctioneer the desired product/service. The buyer posts his request online, invitations to pre-selected vendors are distributed automatically by email; the auction is conducted for a fixed period of time and starts with an opened bid. Interested sellers place their bids, and the buyer can compare prices, qualifications, and other factors quickly. Thus, time between creating the request and choosing the vendor is considerably shortened. Then the sellers bid successively until the end of the event. The seller with the best bid is the winner – usually the lowest bid of the auction - and the buyer purchases the product/service from him. After that, the buyer pays the auctioneer

an agreed percentage of the purchase price as commission [CSIRO2003]. These are also called market-makers because they match customers and suppliers in electronic marketplaces, create the rules, supply services, market and bid analysis, help the buyer to issue specification documents for lots classification.

2. Benefits and risks of reverse auctions

Reverse auction may make it possible to automate some buying and selling processes. Some of the buy-side benefits for using this type of auction are as follow: decreased negotiation cycle, creation of new markets, process improvement, decreases costs in all stages of the purchasing process, companies are able to control parts inventories more effectively, faster return of investment. There are also some of the supply-side benefits: increased visibility of the contract award process, market knowledge, a positive image. [Manaberi2005].

Although there are a lot of benefits of the price-only reverse auction, it still has a major limitation: in non-commodity markets the sellers are loath to compete on price alone. In order to make reverse auctions viable in non-commodity settings, it is necessary to renounce at the restriction on price, while there is a series of factors that need to be satisfied.

Reverse action is a strategic tool used for industry procurement based on an attractive technological solution that allows cost reduction. Buyers, sources and procurement agents make their requests for service to a wider audience by combining reverse auctioning with matching engines and marketplaces. [Sanborn2001]. Price reductions are not the only benefit of reverse auctions to buyers. These auctions also save time because vendor selection is made much faster. Businesses would send out requests for proposals to all potential vendors, wait for them to respond with their proposals, shift through all of them and then make a decision.

The advantage of reverse auction toward sealed bid is that buyers get a lower price, while sellers can tailor their bids to the bidding process.

Only certain procurement activities can benefit from the cost and time savings. Bulk purchases, purchases of goods that are manufactured based on an agreed upon standard also work well for reverse auctions because buyers do not have to be concerned about the quality of the goods since they should all be created equal. Reverse auctions also work better when a buyer will be dealing with a foundation of familiar vendors. If the buyer's needs can only be supplied by a small number of firms, then reverse auctions will not have as a result price reduction. The price in reverse auctions is driven down by competition, so if only limited competition exists, then the price will not decrease enough to save the buyer a substantial amount of money. Reverse auctions don't work well for products or services when large numbers of extra services, such as warranty, are necessary as an important part of the purchasing agreement.

The incorrect implementation of reverse auction can put the relationships with trading partners at risk.

The main ***benefits*** of reverse auction are [spb2006]:

- utilizing of a new procurement option;
- keeping up with changes in technology and business practices;
- achieving a competitive market price, and substantial cost savings through dynamic and real-time trading. Cost savings could be the result of increasing competition, or optimizing market conditions such as surplus inventory in the market place or excess production capacity;
- gaining better knowledge of the market;
- allowing bidders multiple opportunities to offer a price;
- reducing the procurement time cycle and increasing efficiency through real-time process;
- opening up the market by improving access to regional suppliers;
- reducing administrative and transaction costs for both buyer and supplier (less paperwork).

In addition, suppliers have indicated that the reverse auction process is transparent and fair, as all bidders have knowledge of the bids being offered, and suppliers have a good idea if they have been successful at the close of the reverse auction period.

The possible ***risks*** of using the reverse auctions are [spb2006]:

- reverse auctions are new process with *no proven history* – companies should be prudently when using reverse auctions;
- reverse auctions *focus predominantly on price* – reverse auctions have a limited ability to deal with non-price based selection criteria such as economic, social and environmental factors. Also reverse auctions do not evaluate the financial capability and capacity of suppliers to meet procurement requirements. The use of pre-approval, pre-qualification or two-stage procurement processes overcomes these risks. Suppliers should be pre-assessed on the necessary non-price relevant criteria. Reverse auctions then become the final stage, where price is the final determinant;
- *unsuitability for purchase* of services or products with 'value added' features – the evaluation of different value adding services is difficult where the focus is on price. It is also difficult to compare and evaluate solutions, especially if there are innovative solutions offered. Reverse auctions should be used to procure products that are readily defined, and where price can be the primary or final determinant in the selection process;
- *risk of supplier collusion* – if bids are visible to all suppliers, suppliers may not offer their best price, especially if there is insufficient competition. Buyers must have pre-auction estimates, and must ensure a suitable level of competition;
- *possible 'bid shopping'* – bid shopping is the trading off of different tenderers' prices against one another in an attempt to obtain lower prices. Aggressive pricing may increase the risk of buying 'seconds' or 'dumped goods' from local and overseas suppliers. Terms and conditions must require that the quality of goods be certified before payment, and stipulate that dumped goods are not acceptable;
- reverse auctions may *favour large organizations* – there is a perception that large suppliers, which have surplus stocks, production capacity or financial capacity, and which have the necessary infrastructure and skills to participate, have an advantage in reverse auctions. There is also the possibility of cutting out the layer of supplier (middleman) between the manufacturer and the retailer. Reverse auctions may be potentially damaging for local or regional retailers and SMEs;
- *disadvantaging suppliers with infrastructure and Internet limitations* and smaller suppliers who do not have the resources to respond to this online form of market. Version 2 Page 9 Reverse Auctions;
- *reliance on information technology* – there is a risk from technical failure. Reverse auctioneers must have back-up systems and contingency plans.

3. The reverse auction process

The reverse auction process evolves as follows [spb2006]:

1. *define the requirements;*
2. *determine a reverse auction price estimate* – the estimate should reflect the organization best assessment of the current market value for the goods;
3. *identify and select potential suppliers* – a greater number of suppliers invited to participate in the reverse auction will create a greater level of competition and hence better prices for buyers;
4. *notify suppliers of requirements, terms and conditions* – advance notice should stipulate: the procuring organization, all requirements, including any delivery requirements, the date, time and location (website) of the reverse auction, the number or name that identifies the procurement (i.e. lot number), all terms and conditions of the reverse auction process, all terms and conditions of the purchase, including the obligations of buyers and suppliers, the basis on which offers are evaluated and selected, that the final decision rests with the organization;

5. *conduct the reverse auction* – the greater the value of the procurement the greater should be the duration of the auction;
6. *select the successful supplier and place the order* – the terms of the reverse auction should always indicate that the final decision to accept the offer rests with the organization;
7. *conduct post-auction evaluation* – this should refer to the difference between the pre-auction estimate and the auction prices, the performance of the reverse auction service provider.

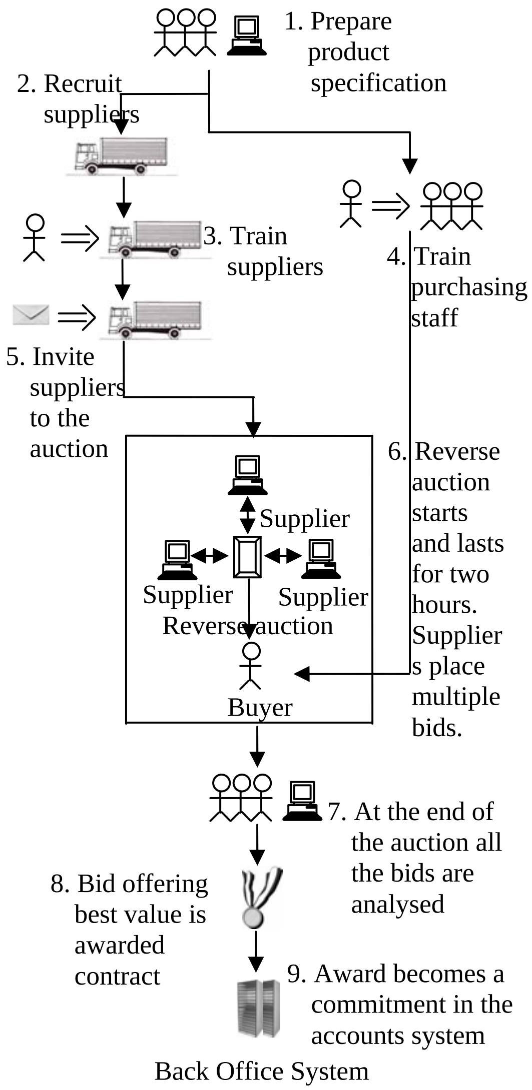

Fig.1. The reverse auction process

4. Electronic reverse auction rules

The electronic reverse auction can be seen as a mediator in negotiation of market-based exchanges. The reverse auction accepts messages in the form of bids (these bids express a willingness to exchange some quantities of resources for a specified monetary value). There are three basic activities in the reverse auction process [Wurman2002]:

- *Handle bid request* – the messages agents send their offers (at any particular time there is one active offer). The bids can be replaced by submitting new bids. In case of a withdraw request, one active bid turns into a null bid.
- *Compute exchanges* - generally, the outcome of an reverse auction is a set of exchanges. The process of computing exchanges is also known as clearing because it doesn't leave any possible exchanges among the remaining bids.
- *Generate intermediate information* – in order to guide bidders, the reverse auction can reveal information about the state of bidding during the event.

Bidding rules – are used in determination of semantic content messages, the authority to place certain types of bids and admissibility criteria for submission and withdrawal of bids.

If the reverse auction takes place for a single item, it is very easy to establish the content of a bid, because it is given by the price. If there are multiple indistinguishable goods/services for sale, from a bid compactness and winner determination complexity perspective, it is recommended to represent these goods as multiple units of a single item, rather than multiple items [Sandholm2001].

In case of multiple units of multiple items, the bid doesn't specify only the price, but also a schedule of prices and quantities indicating the amount of the resources that the bidder is willing to buy/sell at every price. Such auctions are called *multi-unit combinatorial auctions* (in this type of auction, bidders may bid for arbitrary combinations of items). If the bid has a complex expression, it can be simplified by division. Because of the need to trade multiple, inter-related goods or services, this type of auction is useful in business-to-business electronic commerce [Dung2002].

The advantage of combinatorial auctions is given by the fact that the bidders are given the possibility to express their preferences. That is important when items are complements [Jozefiak2007].

5. Clearing problem in reverse auction

Although from an economic perspective the multi-unit multi-items auctions (combinatorial auctions) have a series of benefits, there are also some disadvantages. These disadvantages rise from the small number of the computationally clearing algorithms.

The clearing algorithms, also called the winner determination algorithms, determine a function of the bids made, using [Dung2002]:

- the prices;
- the quantities;
- the trading partners.

In the electronic reverse auction, the players are: one buyer and multiple sellers. We will discuss the clearing problem related to the multi-unit combinatorial reverse auction. So, we consider the following notations:

- 1 buyer;
- n – the number of the bidders;
- $a_1, a_2, \ldots, a_n$ – the bidders;
- m – the number of items (goods or services);
- 1, 2, …, m – the items (goods/services).
- $(q_1, q_2, \ldots, q_m)$ – the auctioneer's demand;
- q_j , $j=\overline{1,m}$ – represents the quantity of item j that the auctioneer needs;
- u_i^j, $i=\overline{1,n}$, $j=\overline{1,m}$ – the maximum quantity of item j that the seller a_i can provide;
- N – the set of natural number;
- R – the set of non-negative real numbers.

Knowing these notations, we can introduce the *supply function* as the price function of the items that each bidder is willing to sell. Thus, the supply function for one bidder i is a mapping P_i:

$$P_i : N^* \to R,$$

where:

$P_i(r_1, r_2, \ldots, r_m)$ – price offered by bidder i for the items $(r_1, r_2, \ldots, r_m)$;

r_j – quantity of item j

$$(r_j \in N, r_j = \overline{0, u_i^j}; \forall\ j = \overline{1, m}).$$

The price function satisfies two properties [Dung2002]:

- *Discount* – this property suggests that is an advantage to buy any combination of two packages of items than buying the two bundles separately because the price in the first case is smaller than in the second. Using the supply function, this property become:

$$\text{If } r_j + s_j \le u_i^j, \forall\ j = 1, m \text{ then}$$
$$P_i(r_1+s_1, r_2+s_2, \ldots, r_m+s_m) \le \quad (1)$$
$$P_i(r_1, r_2, \ldots, r_m) + P_i(s_1, s_2, \ldots, s_m)$$

- *Free disposal* – this property says that if one package has more units of each items than another package, the former is more expensive than the latter. Using the supply function, we can write:

$$\text{If } 0 \le r_j \le s_j \le u_i^j, \forall\ j = 1, m \text{ then}$$
$$P_i(r_1, r_2, \ldots, r_m) \le P_i(s_1, s_2, \ldots, s_m). \quad (2)$$

Using the notations above and the supply function, we can define the bid (ask) as a tuple [Sandholm2001]:

$$A_i = < (u_i^1, u_i^2, \ldots, u_i^m), P_i(r_1, \ldots, r_m) >.$$

We can also give the problem definition [Josefiak2007]:

q_1	…	q_j	…	q_m

u_1^1	…	u_1^j	…	u_1^m		P_1
…	…	…	…	…		…
u_i^1	…	u_i^2	…	u_i^m		P_i
…	…	…	…	…		…
u_n^1	…	u_n^2	…	u_n^m		P_n

The supply function is in deep relation with the *supply allocation*. This can be defined as a tuple $\{r_i^j\}, i=\overline{1,n}, j=\overline{1,m}$ such that the auctioneer buys r_i^j units of item i from each agent a_i [Dung2002].

Knowing these notions and definitions, we can define the clearing problem: *let's find a supply allocation* $\{\alpha_i^j\}, i=\overline{1,n},\ j=\overline{1,m}$ *that satisfies the demand and minimizes the cost*. In other words, the quantity of each item that the auctioneer buys from all bidders is not less than the auctioneer's demand for that item and total price of all units of all items supplied should be as small as possible. The winner determination problem is to label the bids as winning or losing so as to minimize the auctioneer's cost under the constraint that the auctioneer receives all of the units of items that he is asking.
Using the notations above, we have:

$$\sum_{i=1}^{n} \alpha_i^j \geq q_j,\ \forall\ j=\overline{1,m} \qquad (3)$$

$$\sum_{i=1}^{n} P_i(\alpha_i^1, \alpha_i^2, ..., \alpha_i^m) \to \min. \qquad (4)$$

If the sellers are not willing to keep any units of their winning bid and the buyer is not willing to take extra units, in the third formula (3), an equality is used in place of the inequality [Sandholm2001].

6. Clearing problem in multi-unit combinatorial auction

If we approach the clearing problem for multi-unit combinatorial auction, we need to add an additional assumption regarding the price functions: there exists a number $K > 1$ such that for any package of items, if we replace d units of any item with $K \cdot d$ units of any other item, then total price of the new package of items will be more expensive or equal to the price of the old package:

$$P_i(r_1, ..., r_j + d, ..., r_k, ..., r_m) \leq$$
$$\leq P_i(r_1, ..., r_j, ..., r_k + Kd, ..., r_m), \qquad (5)$$
$$\forall\, 1 \leq j, k \leq m,\ j \neq k.$$

Starting from this assumption, [Dung2002] demonstrates some lemmas:

- for any package of items, if we replace d units of any item with d units of any other item, then the total price of the new package is not bigger than K times the total price of the old package:

$$P_i(r_1, ..., r_j + d, ..., r_k, ..., r_m) \leq$$
$$\leq K \cdot P_i(r_1, ..., r_j, ..., r_k + d, ..., r_m), \qquad (6)$$
$$\forall\, 1 \leq j, k \leq m,\ j \neq k.$$

- for any two packages, if the total number of units of the first package is not bigger than the total number of units of the second package, then the total price of the first package is not bigger than K^{m-1} times the total price of the second:

If $\forall$ i = $\overline{1,n}$; $\forall\ r_1, ..., r_m, s_1, ..., s_m$:

$$\sum_{j=1}^{m} r_j \leq \sum_{j=1}^{m} s_j$$

then $\qquad (7)$

$$P_i(r_1, r_2, ..., r_m) \leq$$
$$\leq K^{m-1} P_i(s_1, s_2, ..., s_m)$$

- for any two packages, if the total number of units of the first package is not bigger than the total number of units of the second, then the average unit price of the first package is not smaller than $(2K^{m-1})^{-1}$ times the average unit price of the second package:

If $\forall$ i = $\overline{1,n}$; $\forall\ r_1, ..., r_m, s_1, ..., s_m$:

$$\sum_{j=1}^{m} r_j \leq \sum_{j=1}^{m} s_j$$

then $\qquad (8)$

$$2K^{m-1} \cdot \frac{P_i(r_1, r_2, ..., r_m)}{r_1 + r_2 + ... + r_m} \geq$$
$$\geq \frac{P_i(s_1, s_2, ..., s_m)}{s_1 + s_2 + ... + s_m}$$

Trying to solve the clearing problem of the reverse auction, we can meet two situations:

- the clearing problem has no solution;
- the clearing problem has a solution, corresponding to the following case: the bidders can supply the units that the auctioneer demands and it can be found an allocation. The solution supplies exactly the number of units that the auctioneer demanded.

If the clearing problem has a solution, this can be found using the clearing algorithm. This algorithm uses the (1)-(8) formulas and consists of three steps that are consecutively repeating.

Next, we present the clearing algorithms for the multi-unit combinatorial reverse auctions (CAMUCRA):

CAMUCRA. Repeat the steps:

- For all i, j such that $u_i^j > q_j$, set
$$u_i^j = q_j;$$
- Find the bidder a_k such that:
$$\frac{P_k(u_k^1, u_k^2, ..., u_k^m)}{u_k^1 + u_k^2 + ... + u_k^m} \text{ is minimal,}$$
then select a_k to provide all its units
$$(u_k^1, u_k^2, ..., u_k^m).$$
- Repeat the steps with the new set of bidders $A \setminus a_k$ and demand
$$q_j^{new} = q_j - u_k^j.$$

We now explain the three steps of the CAMUCRA:

- *Step 1* – In order to consider only quantities that are not bigger than the demand, the supply function is truncated (this is necessary to minimize the total price. The auctioneer doesn't need to buy more units than its demand, this is because the price function satisfy the free disposal property as we could see in (2));
- *Step 2* – In the second step, it's been considering all the biggest packages offered by the bidders and it's been choosing the package that offers the lowest average unit price.
- *Step 3* – In the third step, the set of bidders A and the auctioneer's demand are updated. If the set of bidders was (at the second step):
$$A = \{a_1, a_2, \ldots, a_{k-1}, a_k, a_{k+1}, \ldots, a_n\},$$
after we selected a_k, the set of bidders become:
$$A = \{a_1, a_2, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n\}.$$

If the quantity of item j that the auctioneer needs (q_j) was partially satisfied by the the maximum quantity of item j that the seller a_k (selected at the second step) can provide (u_k^j), then the quantity of item j that auctioneer wants to buy (q_j^{new}) will be the difference between q_j and u_k^j.

One of the main issues in auction theory is the performance comparison of different auction formats and the performance of the algorithms used in clearing problem [Ghebreamlak2006].

In CAMUCRA case, each step requires $O(n)$ time to find the smallest element: $\frac{P_k(u_k^1, u_k^2, ..., u_k^m)}{u_k^1 + u_k^2 + ... + u_k^m}$, $k = \overline{1, n}$. That is, each step has $O(n)$ complexity. As there are n steps, the complexity of CAMUCRA CAMUCRA is $O(n^2)$ [Dung2002].

The solution of this algorithm is within a bound $b = 2n \cdot K^{m-1}$ from the optimal. If $P_n(O)$ is the optimal total price and $P_n(S)$ is the total price of the solution of the algorithm. Then:

$$\frac{P_n(S)}{P_n(O)} \leq 2n \cdot K^{m-1}, \qquad (9)$$

where K is the same as in (8). Therefore, the solution is closed to the optimum total price with a very small error.

7. Conclusions

In this paper we presented the main features of the reverse auctions as they are the most frequently type of auction used for e-procurement activities in the B2B environment. Next, we described the benefits, but also the risks of this type of electronic auction. We indicated the phases of a reverse auction and its rules. One of the most important issues in any auction is the winner determination, that's why we analyzed the clearing problem in a multi-unit combinatorial auction. Afterwards, we described the clearing algorithm for the multi-unit combinatorial reverse auctions, the properties and the steps of this algorithm. The conclusion is that it provides a flexible and efficient tool that enables us to benefit from the potential of the combinatorial auctions. In our further work, we intend to develop the algorithm as well as to obtain the best allocation solution and to apply it in real auction scenarios.

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