The Franchise Deductible Policy

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The paper presents applications of credibility theory dealing with real life situations, and implemented on real insurance portfolios. Though more examples could be given, we limit ourselves to the introduction of a franchise deductible policy. In this example we try to demonstrate what kind of data is needed to apply credibility theory. The example shows that credibility theory is really a useful tool-perhaps the only existing tool-for such insurance applications.
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Introduction

All numerical results in this paper were obtained using the software package CRAC 2.0., which use Jewell's hierarchical model. The hierarchical credibility model can be applied to solve quite a number of practical insurance problems. The one studied here is the problem of the franchise deductible policy in credit insurance, but the same philosophy and ideas can be applied for franchise deductible policies in any type of insurance.

1. Problem definition and example

The main practical question is: what is the premium reduction in case a franchise deductible of a% of the commercial premium is introduced? So, the question to be answered in this paper is the following: which discount on the risk premium can be given by introducing a deductible. Let us illustrate this problem by means of a specific example in credit insurance. So, a practical example will be given to illustrate the possibilities of credibility. Assume that:

(i) an insured company pays a (commercial) premium of 100.000 (in some currency), which covers all losses, without franchise deductible;

(ii) the risk premium (commercial premium minus administration costs and safety loadings) equals 60.000 (60% of the commercial premium: $\frac{60}{10^2}\cdot 10^5 = 6\times 10^4$);

(iii) administration costs remain unchanged upon introducing a franchise deductible.

When the insurance company introduces a franchise deductible of a% = 10% of the commercial premium, it will normally be possible to quote a lower premium than 100.000.

In this application we will calculate the discount, to be expressed as a percentage of the risk premium. A typical solution of this problem might look as follows: by introducing a franchise deductible of a% = 10% of the actual commercial premium (of 100.000), the risk premium will decrease by 15%. For our practical example this reads as follows: by introducing a franchise deductible of 10.000 ($\frac{10}{100}\cdot 100.000 = 10.000$), the risk premium decreases from 60.000 (see (ii)) to 51.000 (because: 60.000-$\frac{15}{100}\cdot 60.000 = 60.000 - 9.000 = 51.000$) and the commercial premium (given assumption (iii)) from 100.000 to 85.000 (because: 100.000-$\frac{15}{100}\cdot 100.000 = 100.000 - 15.000 = 85.000$).

Since assumption (iii) is not fulfilled in practical situations and since cost structures can differ from one insurance company to another, we prefer to work on basis of risk premiums. When, for instance, administration costs and other costs also decrease by introducing a franchise deductible from 68,6% to 40%, the final commercial premium will no longer be 85.000 but 71.400 (68,66-40 =

28,6; $100.000-\frac{28,6}{100}=100.000-28.600=71.400$).

2. Mathematical definition of the problem

Let us introduce some variables, in order to define the problem more mathematically:
X = the original claim size;
a = the percentage of the commercial premium giving the deductible;
p = the commercial premium;
R_p = the risk premium (commercial premium minus administration cost and safety loadings).
The question that arises is to find an estimate for:

$$Y = 100(x-ap)_{+}R_p \qquad (1)$$

The ratio of the excess risk "$(X-ap)_{+}$" divided by the pure risk premium "R_p" is considered, because the new loadings of the franchise deductible policy can be different from the original loadings "X", as illustrated in the previous subsection. To obtain the result as a percentage of the risk premium, it is multiplied by 100.

3. Calculation of the risk premium; determination of sectors

Clearly, the first problem we have is to determine the pure risk premium, therefore to evaluate x, where R_p = x%p. In general in an insurance company this figure is not known explicitly. This problem, too, can be solved by means of credibility theory, as will be explained later on.
First, we will give some more information on the model used. Using the semi-linear hierarchical model, the total portfolio is split into P different sectors; sector *p* contains k_p individual policies. In this example 14 sectors are defined, on the basis of two criteria:
-territory: domestic (sectors 1, 2,…, 7) vs. abroad (sectors 8, 9,…, 14);
-average turnover: 7 different groups (1/8 = small,…, 7/14 = immense).
Using credibility we will compute:
-a global estimate for the entire portfolio, denoted by m;
-a result for each sector *p* (*p* = 1, 2,…, P) denoted by N_p^a;
-a result for each policy j (j = 1, 2,…, k_p) in sector *p*, denoted by M_{pj}^a.
The subdivision in categories is arbitrary and based on the feelings of the actuary of the company. Other subdivisions are possible. Note that even if the original subdivision in categories is not correct, credibility theory will adjust the final results. If a subclass is not homogeneous enough, the measure for the variability (s^2) will be relatively large such that the credibility factor will be relatively low. But when some of the subclasses are large enough to be considered on their own, credibility theory will reveal this by obtaining a credibility factor equal to one.
To solve the preliminary problem of determining the loadings in the commercial premium the model is first solved for a = 0. With a = 0, obviously Y as in (1) reduces to $100X/R_p$.
Suppose that the insurer is convinced that before introducing the deductible the global premium income is sufficient to cover both costs and risk. Then an overall estimate m of Y for the entire portfolio should give m = 100. In a top-down approach we start from the hypothesis that the total portfolio without deductible is in financial equilibrium. This does not mean that no premium deviations per sector (or per individual policy), both positive and negative, exist.

4. Numerical results for the franchise deductible policy

In this example we start performing credibility calculations for a = 0 and with R_p = 100%p, which means that the total commercial premium is considered as risk premium.
As weights we use the average turnover (AT) of the company. The credibility calculations are evaluated iteratively to reach the result m = 64,21%. In order to arrive at m = 100% we have to choose R_p = 64,21%p. This means that the risk premium, in case of global financial equilibrium of the portfolio, equals 64,21% of the total commercial premium. A summary of the resulting discounts for a = 0%, 25%,…, 150% is given in the two tables below. Table I gives global results and results per sector. The first column contains the

number of contracts in each sector. The upper half of the table gives the global result m and the results per sector, the lower half the corresponding credibility factors z_p, expressed as percentages. In table II, an example of individual results (per policy) is given. Both tables will be discussed in some detail in the following sections.

5. Validation of an existing tariff structure

Let us first concentrate on the results per sector with a = 0%. This means that we do not consider a deductible and therefore examine an existing tariff structure, under the assumption that a global financial equilibrium exists. As expected, it is found that in spite of this global financial equilibrium, premium deviations per sector, both positive and negative, exist. From Table I we notice that sectors 1, 6, 7, 8, 9 and 11 should have a premium increase varying from 0,09% for sector 11 to 18,4% for sector 8, whereas all other sectors could benefit from a premium discount, varying from 1,42% for sector 14 to 12,21% for sector 10, even without introducing any deductible. The discounts of - 18,4% and + 12,21% in sectors 8 and 10 seem remarkable at first sight. Discussing these results with people of the insurance company concerned, we learned that they had intuitively felt that something was wrong with the premiums of these classes, and the numerical results were an excellent proof of their intuition.

Let us now investigate in more detail the credibility weights z_p of the different sectors for a = 0%. The calculated weights confirm the following intuitive feelings:

1) small subclasses have a smaller weight than those with many policies. *Example*: in sector 1, 318 policies lead to weight of 24,2% while in sector 8, 813 policies have a weight of 33,4%.

Both sectors contain policies with a comparable average turnover. Since there are more policies in sector 8, our intuitive feeling that sector 8 should have a more important weight is confirmed.

2) Sectors that contain “important” policies have a –relatively- higher weight. In this case, the most important policies are those with the highest average turnover (sector 7 and 4). *Example*: sector 7 with 14 policies has weight 11,9% , and sector 4 has 296 policies and weight 58,8%. Sector 7 has 20% of the weight of sector 4, but contains only 4,7% of the number of policies of sector 4. This is due to the fact that the policies in sector 7 are more important than those in sector 4.

6. Credibility calculations for the franchise deductible policy

Having determined the risk premium, we can concentrate on the basic problem of the influence on the commercial premium of introducing a deductible. Interpretation of the results in Table I is straightforward.

From the third line in the table we learn that the global discount m (for the portfolio as a whole) increases slower for higher values of a.

This is as it should be. In practice, however, we often saw tariffs where this principle was uniformly distributed.

A table as Table I with the results per sector has proved to be of great importance in practical insurance business. It is well known that it is sometimes easier to impose a deductible than to increase the commercial premium. From the table we learn for instance that, for sector 8, instead of a premium increase of 18,4% (when a = 0%) we could introduce a deductible of a = 50% of the commercial premium. This operation will also lead to an equilibrium for this sector.

Often insurers want to impose deductibles not for a sector as a whole but for individual policies. Especially in types of insurance where a limited number of contracts generates a huge premium income, such an individual tarification might be desirable. In some cases it is even indispensable to have the possibility to construct a tariff on an individual basis, because important clients often require a special premium settlement.

Table II gives an example of a possible table of individual premiums. Only a very limited number of the approximately 3700 contracts in the portfolio is included in this table. The first and second column contain policy number and sector number, the third column gives the average turnover (AT). The next seven columns give the calculated discounts for different values of a.

Table I

Discount percentages for franchise deductible policy; global and sector results

a =	0%	25%	50%	75%	100%	125%	150%
# iterations	7	15	20	15	20	14	20
m =	-0,18%	19,92	34,42%	44,53%	52,40	57,92	62,68
# sector							
318|N1	-3,88	14,00	26,38	35,57	42,83	48,46	53,36
763|N2	4,78	24,65	38,18	47,85	55,29	60,76	65,34
549|N3	4,95	27,41	42,93	53,71	61,69	67,65	72,45
296|N4	2,01	26,62	44,93	56,63	65,32	71,43	76,50
151|N5	3,92	28,77	46,14	57,53	65,83	70,97	74,97
38|N6	-8,02	10,69	26,44	38,65	47,85	55,07	60,36
14|N7	-5,12	14,01	31,89	46,83	58,38	66,13	72,95
813|N8	-18,40	-7,37	-0,70	4,59	9,29	13,68	17,97
301|N9	-4,82	9,95	19,69	26,92	32,50	37,24	41,12
207|N10	12,21	32,72	46,63	55,89	63,20	68,59	70,00
127|N11	-0,09	18,88	32,05	40,98	47,91	52,87	56,85
95|N12	5,44	28,55	44,52	55,13	63,67	69,19	74,06
23|N13	3,11	25,79	41,34	50,68	58,49	63,22	67,73
7|N14	1,42	24,24	41,87	52,41	61,31	65,68	70,83
318|z_1	24,20	31,30	37,60	41,60	44,30	45,60	46,20
763|z_2	59,90	69,10	74,80	77,70	79,70	80,30	80,90
549|z_3	64,30	74,10	79,40	81,70	83,50	83,80	84,50
296|z_4	58,80	71,10	77,20	79,30	81,60	81,60	82,70
151|z_5	50,10	65,60	73,00	74,90	78,00	77,30	79,10
38|z_6	24,20	40,20	49,50	51,20	56,00	54,30	57,60
14|z_7	11,90	23,80	32,00	32,80	37,80	35,10	39,20
813|z_8	33,40	41,50	48,40	52,50	55,20	56,60	57,20
301|z_9	36,40	46,00	53,20	57,10	60,00	61,00	61,80
207|z_{10}	40,20	51,70	59,10	52,50	65,50	66,00	67,20
127|z_{11}	37,90	51,30	59,20	62,10	65,50	65,50	67,10
95|z_{12}	39,50	55,80	64,20	66,30	70,00	69,10	71,40
23|z_{13}	16,30	29,30	37,80	39,30	44,00	42,10	45,60
7|z_{14}	6,20	13,00	18,30	18,90	22,50	20,70	23,50

Let us now compare the results per sector with the individual results. It is clear that, even after splitting up the total portfolio in more homogeneous subclasses or sectors, a certain heterogeneity within each of the subclasses remains. When we take a closer look at the individual results for the policies belonging to sector 8, we see that discount factors range from -6% (see the case: a = 0%) to -26% (see the case: a = 0%), whereas the discount percentage for the sector was -18,4% (see the case: a = 0% from the table I). From tables like this one we can deduce the premium increase or decrease for different values of a on an individual basis. The insurance company-or the insured person-can then choose whether he prefers a premium increase (or for some policies a premium decrease), or a deductible.

Table II

Discount percentages for different a-values

Policy nr.	Sec.	AT	a = 0%	25%	50%	75%	100%	125%	150%
0021011	10	10	45	50	59	67	72	77	79
0031010	8	0	-17	-6	0	5	10	14	19
0070016	8	2	-21	-7	0	6	11	16	20
0081005	8	2	-26	11	-3	3	9	14	19
0141010	10	6	17	37	52	63	69	74	77
0161008	9	3	11	18	26	34	38	43	46

0431003	9	3	3	14	23	32	37	-43	46
0471010	8	1	-12	-4	2	8	12	17	20
0581010	8	22	-19	-8	-1	5	10	14	19
0621006	8	0	-17	-6	0	5	10	14	19
0651003	8	1	-19	-7	0	6	11	16	20
0681000	8	1	-12	-4	2	8	12	17	20
0710029	7	1473	-122	-83	-44	-5	25	54	70
0881002	10	6	25	41	53	63	69	74	77
0891001	8	0	-17	-6	0	5	10	14	19
0950016	10	6	30	42	54	63	69	74	77
1141020	8	2	-6	-1	5	11	14	19	23
1211002	10	6	35	44	54	63	69	74	77
1260018	6	70	41	58	68	80	83	87	87
1271001	9	3	-27	-3	8	14	22	25	31

We can even go further and say that the insured himself can determine which premium he wishes to pay. The insurance company simply has to determine which value of a corresponds with this premium.

Table II can also be used to find really bad contracts or what we call “outliers”.

The most remarkable case in this example is policy number 0710029, belonging to sector 7. The discount factor for sector 7 without deductible (when a = 0%) is – 5,12% (see the table I), whereas for this individual policy a premium increase of 122% is necessary.

A summary of the resulting discounts for a = 0%, 25%,…, 150% is given in the two tables below.

Conclusions

The example shows that credibility theory can be used in the design of a premium structure for a heterogeneous portfolio. It can also be applied to examine an existing tariff structure. If a portfolio, subdivided into cells, is considered together with a given tariff structure, the credibility calculations can be performed, giving an optimal premium estimate in every cell and also an estimate for the global premium.

Comparing the global premium with the overall credibility premium provides us with an indication of the necessary loading. Applying this loading factor to each of the premiums per cell gives us an estimate to the real market premium gives us a clue on how an existing tariff structure is performing. In addition it gives information on the technical gains and losses in each of the subclasses. So, it enables one to measure the efficiency of an existing tariff.

The fact that it is based on complicated mathematics, involving conditional expectations, needs not bother the user more than it does when he applies statistical tools like SAS, GLIM, discriminant analysis, and scoring models. These techniques can be applied by anybody on his own field of endeavour, be it economics, medicine, or insurance. We give a rather explicit description of the input data for the program CRAC 2.0, used, only to show that in practical situations there will always be enough data to apply credibility theory to a insurance portfolio. The point we want to emphasize is that practical application of credibility theory is feasible nowadays using appropriate software.

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