Market Indexes and the Measurament of Portfolio Performance

Professor Stelian STANCU, PhD.
Department of Economic Cybernetics, Academy of Economic Studies Bucharest

The article presents essential aspects of the market indexes and the measurement of the stocks portfolio performance.
Under this context, we shall present the aggregate simple price index, the geometric index, the arithmetic index (with market capitalization) and the dividends index.
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The characteristics of the developed investment market are the utility and the completeness of the indexes that determine the aggregate changes.
These indexes are dependable of price but they can also depend of other statistics indexes such as the total income or the efficiency.
Some the fundamental questions asked when we define an index are:

- How many components (stocks) shall be used for the index formula and how should we choose them?
- Will the index be representative for the market as a whole?
- Is the index an arithmetic or a geometric mean?
- Are the index components weighted with equal weights and if they are not, how shall they be weighted?
- How often is the index computed?

The answers to these questions are largely determined by the purposes to which we compute the index.
The purposes could be:

- The measurement of the short term exchanges of the market;
- The process of decision taking, respecting the market as a whole, the market sectors as well as of the individual stocks from different market sectors;
- Temporal historic studies;
- Measurement of portfolio performance.

An index permits:

- To formulate the basis for the derived instruments, such as the futures contracts and the options which develop a protection method against the market changes;
- To be constructed with minimum funds and operational costs.

The investor's main objective is to construct and maintain an investment portfolio which offers the best performance. The investors can measure the performance cost of a single activity by comparing the current and the base period price.
So, they can control the relative price, $\frac{P_t}{P_0}$ where P_t is the price at the time t and P_0 is the price at the zero moment, adjusted for any change of the capital, such as the stocks or bonds emission.
But, the stock exchange listed thousands of stocks and is very difficult to imagine the market movement with the strengthen of the single stocks. Further more, the market values of some firms are larger then others and this should be taken into account when we define the index.
We shall study how we define different types of indexes, such as the most common on the capital markets.

1. The simple price aggregate index

Un index with n stocks types can be defined by simple adding the prices for the n components and by dividing to a divisor.
So, the index at the time moment t is:

$$I_t = \frac{k\sum_{s=1}^{n} P_{st}}{D_t} \qquad (1)$$

where: k – the base moment index (100/or 1000); Ps_t – the price of the s component at

the time t; D_t – the divisor value at time t, adjusted for some capital changes or for the changes of the index components, in order to insure its continuity.

The divisor value at the base moment is $D_0 = \sum_{s=1}^{n} P_{S0}$ given different value to k

We shall conclude that $k = I_0$, because

$$I_0 = \frac{k \sum_{s=1}^{n} P_{s0}}{D_0} = \frac{k \sum_{s=1}^{n} P_{s0}}{\sum_{s=1}^{n} P_{s0}}.$$

If there are some capital changes or changes for the index components at the moment t, then the new divisor is $D_t^{'} = \frac{\sum_{s=1}^{n} P_{st}^{'}}{\sum_{s=1}^{n} P_{st}} \cdot D_t$ where: $P^{'}_{St}$ – the stock price (of the s component) immediately after the capital change; P_{St} – the stock price (of the s component) before the change.

We must underline the idea that immediately after the change, the index is equal to the index before the change. This idea is used in order to measure the short term changes of the capital market.

The Industrial Dow – Jones index in the USA and the Nikkey index in Japan are 2 examples for the simple price aggregate index.

The *Dow – Jones Industrial* index is based on 30 prices of the largest and most transactional stocks from the New York Stock Exchange and was first published in 1928. At the beginning there were used a small number of components but, in the last years, the index included some unindustrialized stocks.

The Nikkey Average index is based on the 225 stocks prices from the first category at the Tokyo Stock Exchange and has been published for the first time in 1950. The stocks emissions which don't have a nominal value of 50 yens are converted to values of 50 yens. The stocks selection is used in order to insure an equilibrated balance of the industry sectors and of other sectors, for the market and the whole economy performance.

2. The geometric index

This type of index is obtained by extracting the n order root from the relative prices product of n stocks, multiplied by k, where k shall represent the base index.

$$\text{So: } I_t = k\left[\left(\frac{P_{1t}}{P_{10}}\right)\cdot\left(\frac{P_{2t}}{P_{20}}\right)\cdot.....\cdot\left(\frac{P_{nt}}{P_{n0}}\right)\right]^{\frac{1}{n}} \quad (2)$$

where: Ps_0 – the s component price at the zero time moment (s = 1,……n); Ps_t – the s component price at the time t (s = 1,……n); k – the base index (100 or 1000 points).

A geometric index has the following characteristics:

- The change in the price of one component influences the index.

For instance, a geometric index for 30 shares from which 29 don't change their price for a given time period and the 30th share has a price greater with 34.8%, then the index will be increased with 1% because $(1{,}01 = \sqrt[30]{1{,}348})$;

- Large changes in the single components values don't find themselves in the index with the same impact, as we can see from the previous example;
- It doesn't correspond to a feasible index.

In particular, a portfolio which consists of equal sums at the initial moment of the components in the index will have the same performance as the best portfolio performance on the period from the initial moment.

- If a component decreases to 0, then the index decreases to 0. In the reality, such component which has a main position will be extract from the index and will be replaced with a new component which which doesn't affect the index so much.

The geometric index has unwanted characteristics for most of his possible utilizations. In particular, the index must not be used for measuring the portfolio because it doesn't represent a possible portfolio. A practical advantage of the geometric index is that it is easily adjusted to the capital changes such as the stocks and bonds emissions, by adjusting the *base price* of the single shares.

The Financial Times Ordinary Share Index and *The Value Line Composite Average* are

index examples which represent a possible portfolio.

The Financial Times Ordinary Share Index is based on 30 types of shares with high market value, which cover a large number of enterprises from the British industry and it exists starting with 1935.

The index is computed in every minute of every working day. The index was choused to measure the short terms changes on the British capital market but, today, his role has been taken by the Financial Times – 100SE, index based on weights – market capitalization and includes a larger number of stocks.

The Value Line Composite Average is based on the 1500 types of shares from the NYSE and AMEX - American Exchanges firms and from the extra stock exchange American market. The index is daily computed by multiplying the index from the previous day with the geometric mean of the relative prices of shares from the current day.

3. Arithmetic index (with market capitalization)

This type of index combines the concept of weighted arithmetic mean of the relative price, equivalent with a Laspeyres index with the concept of chain connection which insures the index continuity when there are capital changes or changes of the index components.

A large number of shares are included in the index, which is being influenced by any large change in the price of one of the components.

Before of the first capital or components change, the index reflects the mean performance weighted of the components price, where the weights are market capitalized at the zero time moment.

In this context, the arithmetic index is:

$$I_t = \frac{k\sum_{s=1}^{n} N_{s0}P_{s0}\left(\frac{P_{st}}{P_{s0}}\right)}{\sum_{s=1}^{n} N_{s0}P_{s0}} \qquad (3)$$

where: k – represents 100 or 1000 points and represents the base index; Ns_t – represents the number of shares of type s issued at the moment t; Ps_t – represents the price of one share of type s at the moment t.

If there are no capital changes ($Ns_t = Ns_0$), then we'll have:

$$I_t = \frac{k\sum_{s=1}^{n} N_{st}P_{s0}\left(\frac{P_{st}}{P_{s0}}\right)}{\sum_{s=1}^{n} N_{s0}P_{s0}} = \frac{k\sum_{s=1}^{n} N_{st}P_{st}}{\sum_{s=1}^{n} N_{s0}P_{s0}}$$

because the Laspeyres index for the price is written:

$$I_L = \frac{\sum_{s=1}^{n} P_{st}q_{s0}}{\sum_{s=1}^{n} P_{s0}q_{s0}} \times 100 = \frac{\sum_{s=1}^{n} P_{s0}q_{s0}\left(\frac{P_{st}}{P_{s0}}\right)}{\sum_{s=1}^{n} P_{s0}q_{s0}} \times 100.$$

unlike the Paasche index for the price which is:

$I_P = \frac{\sum_{s=1}^{n} P_{st}q_{st}}{\sum_{s=1}^{m} P_{s0}q_{st}} \times 100$, where q is the index for the quantity.

Generally, in order to allowed the capital changes, the index at time t is computed:

$I_t = \frac{k\sum_{s=1}^{n} P_{st}N_{st}}{B_t}$ where B_t – represents the base for the index at time t.

The value of the base index at the initial time represents the capitalization of the index components at the initial time, $B_0 = \sum_{s=1}^{n} P_{s0}N_{s0}$, which goes to $k = I_0$.

In order to insure continuity, the value of the base index will be discounted any time there is a capital change so the index will be the same.

This is known as the chain connection. We assume there is a change in the capital C_t at time t. The index and the base index before the change are I_t, and B_t.

If after the change we'll consider the new index and the new base of the index as I'_t, and B'_t we'll get:

$$I_t = \frac{k\sum_{s=1}^{n} P_{st}N_{st}}{B_t} \Rightarrow$$

$$\frac{k\sum_{s=1}^{n} P_{st}N_{st}}{B_t} = \frac{k\left(\sum_{s=1}^{m} P_{st}N_{st} + C_t\right)}{B'_t}$$

$$I'_t = \frac{k\left(\sum_{s=1}^{n} P_{st}N_{st} + C_t\right)}{B'_t}$$

$$\Rightarrow B'_t = \frac{B_t\left(\sum_{s=1}^{n} P_{st}N_{st} + C_t\right)}{\sum_{s=1}^{n} P_{st}N_{st}} = B_t + k\frac{C_t}{I_t}$$

$I_t = I'_t$ for continuity.

There are many examples of arithmetic weighted indexes with capitalization, included F.T. - Actuaries All - Share Index (Great Britain), F.T. – SE 100 Index (Great Britain), S&P Standard and Poor 500 Index (USA), The Tokyo Stock Exchange New Index (Japan), The Morgan Stanley Capital International Indices and The F.T. – Actuaries World Indices.

The F.T. - Actuaries All - Share Index is often used in Great Britain for investments decision making and for the measurement of the portfolio performance. The index is daily published based on the mean close prices from the previous day.

The firms are divided into industrial sectors, each sector with its own index. The most part of the managers from the Great Britain uses the index for measuring the performance of their activity, for small and medium firms.

The F.T. – SE 100 Index is an index computed for 100 large firms from the Great Britain. It has been introduced in 1964 in order to take into account the futures index on shares and bonds and it is computed in every minute of the working day.

The S&P Standard and Poor 500 Index is computed based on the prices of the 500 types from the most traded American shares, equal distributed in every sectors.

As for the *F.T. - Actuaries All - Share Index*, his computation begins from groups of industries towards the whole index and this index is used for studying the relative performance of the share from the respective industrial group. This index is generally considered as an index needed for the measurement of the American capital portfolios.

The Tokyo Stock Exchange New Index consists of all listed shares from the first category to the Tokyo Stock Exchange. We compute sub indexes for each industrial group. Even that some of the shares from the index don't have any clear utility, the index is considered as being one of the most suitable indexes for measuring the performance of the Japanese capital portfolios.

The Morgan Stanley Capital International Indices and *The F.T. – Actuaries World Indices* are global indices, most used for the decisions concerning the portfolio allocation and measurement.

The FT – Actuaries All – Index is considered by many countries and regions as a *global index*.

The above indices are used in any currency but they are frequently published in $, £, yens, euros.

We assume that I_t – represents the index in the local currency at time t for a country. So, the index expressed in dollars for that country is: $\$I_t = I_t(D_t/D_0)$ where:

D_t – is the value in dollars of each unit of local currency at time t;

D_0 – is the base value of the index at time t = 0.

The index for that country expressed in other currency can be similarly obtained and any currency index is changed once with the change of a local currency index combined with the fluctuation of the exchange rate.

4. The dividend performance. The dividend indexes.

The classic method to compute the index of the dividend performance according to the index with the capitalization as a weight, is the method that divides the total of paid dividends in the last year (the FT - Actuaries All – Share) to all types of shares (components) that are included in the index computation at the total market value of the shares. So:

$$I_t = \frac{\sum_{s=1}^{n} N_{st}D_{st}}{\sum_{s=1}^{n} N_{st}P_{st}} \times 100 = \frac{\sum_{s=1}^{n} N_{st}P_{st}\left(\frac{D_{st}}{P_{st}}\right)}{\sum_{s=1}^{n} N_{st}P_{st}} \times 100 \quad (4)$$

where D_{st} – represents the dividend per share

of type s at the time t.
In other words, the dividends performance is a mean, weighted by capitalization where the weight is the market capitalization of all types of shares.

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