The Monte Carlo Simulation Technique Applied in the Financial Market

Cornelia MUNTEAN, Faculty of Economic Sciences
West University of Timişoara, Romania,
E-mail: cornelia.muntean@fse.uvt.ro

Monte Carlo simulation is a mathematical technique involving repeated simulation of a system with random sampling from probability distributions of real life processes. The Monte Carlo method is widely applied to large and complex problems to obtain approximate solutions. This method has been successfully applied to problems in physical sciences and, more recently, in economy and finance. Sufficient number of repetitions, called iterations, is required to arrive at a statistically viable result - usually an average value of a parameter. In today's competitive market environment, especially in the financial market, Monte Carlo is applied generally for risk management, in the calculation of Value at Risk (VaR) and Profit at Risk (PaR). However, the method of applying the Monte Carlo technique differs between the financial market and other markets, as for example the electricity market.
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1 Introduction and short history of Monte Carlo simulation technique

The Monte Carlo method provides approximate solutions to a variety of mathematical problems by performing statistical sampling experiments on a computer. This method has been successfully applied to problems in physical sciences and, more recently, in finance. Many difficult financial engineering problems, such as the valuation of multidimensional options, path-dependent options, and stochastic volatility or interest rate options can be tackled thanks to this technique.
Monte Carlo simulation in general is used when analytical methods are either not available or are available but the mathematical procedures are so complex that simulation provides a simpler method of the solution.
The method is called after the capital city of Monaco, because of roulette, a simple random number generator. The name and the systematic development of Monte Carlo methods date from about 1944.
There is however a number of isolated and undeveloped instances on much earlier occasions.
For example[1], in the second half of the nineteenth century a number of people performed experiments, in which they threw a needle in a haphazard manner onto a board ruled with parallel straight lines and inferred the value of PI = 3.14… from observations of the number of intersections between needle and lines. An account of this playful diversion (indulged in by certain Captain Fox, amongst others, whilst recovering from wounds incurred in the American Civil War) occurs in a paper of A.Hall (A. HALL 1873. " On an experimental determination of PI").
According to Eckhardt, Ulam invented the Monte Carlo method in 1946 while pondering the probabilities of winning a card game of *solitaire*[2]. However, Metropolis "attributes the germ of this statistical method to Enrico Fermi, who had used such ideas some 15 years earlier"[3]. *Allegedly, Nick Metropolis coined the name 'Monte Carlo', which played an essential role in popularizing the method*". Winston (1996, p.22) wrote that the term was coined by mathematicians S. Ulam and J. von Neumann in the feasibility project of atomic bomb by simulations of nuclear fission, and they given the *code name* Monte Carlo for these simulations.
The first Monte Carlo paper, "*The Monte*

[1] http://stud4.tuwien.ac.at/~e9527412/index.html

[2] www.riskglossary.com/articles/monte_carlo_method.htm
[3] scienceworld.wolfram.com/biography/Metropolis.html

Carlo Method" by Metropolis & Ulam, was published in 1949 in the *Journal of the American Statistical Association*. Since then, several different areas have been using the Monte Carlo simulations. With the advent of personal computers and the popularization of faster computational machines, the Monte Carlo simulations have been increasing popular as an important alternative for the solution of complex problems.

2. Simulation using Monte Carlo.

There are eight steps involved in simulation using Monte Carlo:

Step 1. Describe system probability distributions

Describe the system and obtain the probability distributions of the relevant probabilistic elements of the system. This step requires intimate familiarity with the system and incorrect assumptions at this point invalidate the rest of the simulation.

Step 2. Decide on the measures of performance.

Define the appropriate measure of system performance. If necessary, write it in the form of an equation. Some examples are: average daily profit, average annual spot prices, average monthly spot prices, lost of load probability, average annual unsaved energy. This depends on the objective. System reliability studies focus on unsaved energy and loss of load indices. Investment feasibility studies generally focus on average energy prices and plant capacity factors.

Step 3. Compute cumulative probability distributions.

Construct cumulative probability distributions for each of the stochastic elements.

Step 4. Assign representative ranges of numbers.

Assign representative numbers in correspondence with the cumulative probability distributions.

Step 5. Generate random numbers and compute the system's performance.

For each probabilistic element, take a random sample.

Step 6. Compute measures of performance.

Derive the measures of performance. Each Monte Carlo simulation run is composed of multiple iterations. The question of the number of iterations required involves statistical analysis. The larger the number of iteration, the more accurate the results, but it takes more time and the cost is higher. This issue concerns what are labeled as stopping rules. The run could be terminated when a desired standard error in the measures of performance is attained.

Step 7. Stabilization of the simulation process.

Simulation begins to represent reality only after stabilization has been achieved. Therefore, we distinguish a start-up period during which the data results are not yet valid. The length of the simulation must be sufficient for the system to reach stability.

Step 8. Repeat steps 5 and 6 until measures of system performance stabilize.

When the differences between each sample's index of measure, say loss of load probability (LOLP), average prices or other index, becomes small or insignificant, then the process is said to have stabilized. The significance level is set typically by management.

Generally, the Monte Carlo procedure involves generating a large number of realizations of the underlying process and, using the law of large numbers, estimating the expected value as the mean of the sample. This translates into following algorithm:

1: for j=1 to N do

2: Simulate sample paths of the underlying variables (asset prices, interest rates, etc.) using the risk neutral measure over the time frame of the option. For each simulated path, evaluate the discounted cash flows of the derivative C_j

3: end for

4: Average the discounted cash flows over the sample paths $\hat{C} = \frac{1}{N}\sum_{j=1}^{N} C_j$

5: Compute the standard deviation

$$\hat{\sigma}_{\hat{C}} = \sqrt{\frac{1}{(N-1)}\sum_{j=1}^{N}\left(C_j - \hat{C}\right)^2}$$

We note that the standard deviation of the Monte Carlo estimation $\hat{C}$ decreases at the order $O(1/\sqrt{N})$ and thus that a reduction of a factor 10 requires an increase of the number of simulation runs N of 100 times.

3. Financial Market Model

Where the Monte Carlo simulation model in the financial commodities market differs from other markets is perhaps in the determination of the relevant probabilistic elements of the respective systems and their corresponding probability distributions.

In the financial markets, the relevant elements are typically the commodity/share prices, the interest rates and the exchange rates. The commodity and share prices are generally determined by price expectations of the masses of investors and traders in the market system. The large number of traders (investors) generally results in a common probabilistic behavior, that is, one that follows a normal and lognormal distribution. This investor behavior is affected by economic conditions and sometimes actual real life news events but in general, not affected by real time physical events such as say, a generating unit falling out of service.

In short, the financial market model can be described by those elements such as the commodity/share price which have a past history and known probabilistic behavior.

4. Relevant Elements of the System. Conclusions

In other markets, as for example in the electricity market, the relevant elements are different.

Here the prices, unlike in most commodities in the financial market, exhibit a strong coupling with the condition of the physical power system. These prices are not as dependent on investor behavior as they are dependent on the level of capacity reserves in the system, the system demand and level of congestion in the network. Because of this strong physical coupling, it is unwise to represent that price behavior as simply an equation devoid of relationship with the physical system.

Although, it is possible to represent the historical price behavior in an equation form, encapsulating its mean reversion and jump behavior within the equation parameters, this necessitates gross assumptions and simplifications. There may be some applications of this approach but only within a limited simulation time frame. Its application to long term financial market forecasting is questionable.

A better way to model the price behavior is to include the relevant elements that affect it within the simulation. These elements are the demand, the supply, the market and power system and the bidding behavior[4]. Probabilistic distributions are then applied to both supply and demand. For the supply, the generator unit random forced outages are described by a special kind of distribution called a Weibull distribution. This type of distribution is applied to the mean time for failure and mean time to repair of the type generating unit. Price is determined by taking these different elements and their interaction into account, thereby avoiding the need for gross assumptions and simplification.

In today's competitive financial market with its inherent uncertainties and complexities, Monte Carlo simulation can be used to forecast future prices and market behavior. Understanding the technique and having the right tools can spell the difference between success and failure in short term and long term trading and investing.

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