The Use of Gravity Models for Spatial Interaction Analysis

Prof. Daniela-Luminiţa CONSTANTIN PhD.
Academy of Economic Studies, Bucharest

Analogously to the gravity law in physics, gravity models in regional economics state that the interaction between two centers is in direct proportion to their size and in inverse proportion to the distance (at a certain power) between them. This paper aims to emphasize the role of gravity models in various fields of spatial interaction analysis, focusing on market area boundary, commodity flows, migration.
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The gravity models provide a flexible approach for the analysis of spatial interactions between spatially separated nodes, being applied in a wide variety of studies, such as those devoted to migration, commodity flows, traffic flows, residence-workplace trips, market area boundaries, etc.

In general terms, the gravity models state that the interaction between two centers is in direct proportion to their size and in inverse proportion to the distance (at a certain power) between them. Thus, if the centre size is represented by population number, the gravity model may be written using the following function: $\mathbf{I_{ij}} = \mathbf{G} \cdot \frac{\mathbf{P_i} \cdot \mathbf{P_j}}{\mathbf{d_{ij}}^{b}}$, where: $\mathbf{I_{ij}}$ = the interaction between the two centers, i and j; $\mathbf{P_i}, \mathbf{P_j}$ = the size (population) of i and j; $\mathbf{d_{ij}}$ = distance between i and j; $\mathbf{G}$ = a constant (proportion factor).

This paper discusses a series of relevant cases for gravity models' employment, proving that these models are really useful to underlying various domains of regional policy and decision-making process at this level.

A gravity model to determine the market area boundary. The most frequently used model for this purpose is the so-called Reilly's law of retail gravitation, stating that a centre tends to attract retail trade from an individual customer located in its hinterland in proportion to its size (as measured by population) and in inverse proportion to the square distance between them.

If X and Y are two competing centers the market area boundary between them will be the point Z, where the relative pull of the two centers is equal:

$$\frac{P_X}{d^2_{XZ}} = \frac{P_Y}{d^2_{YZ}} \quad (1)$$

where: P_X, P_Y = population of centre i, respectively j; d_{XZ}, d_{YZ} = distance between X and Z, respectively between Y and Z.

Assuming that **r** is the distance between X and Y, **s** is the ratio P_X / P_Y and **z** the distance between X and Z (unknown), the equation (1) may be rewritten as

$$s = \frac{z^2}{(r-z)^2} \quad (2)$$

Rearranging and solving for z, is obtained

$$z = \frac{\sqrt{s}}{1+\sqrt{s}} r \quad (3)$$

In particular, if s = 1 (i.e. X and Y are of the same size), $z = \frac{1}{2}$ r, meaning that the boundary is at equal distance from X and from Y.

Although (or rather because) Reilly's law is an empirical rather than a theoretical construct, it has been applied reasonably well in practice. The population of each centre serves as a substitute for the variety of goods produced by that centre, whereas the square of distance assumption is a way of adjusting the geographical distance for the costs and disutility of travel.

Doubly constrained gravity models for commodity flows analysis. The simplest type of spatial interaction described by a commodity flow model is

$$\mathbf{F_{ij} = a \cdot D_j \cdot f\left(c_{ij}\right)} \qquad (1)$$

where: $\mathbf{F_{ij}}$ = commodity flow from i to j; $\mathbf{D_j}$ = demand at j; $\mathbf{c_{ij}}$ = transport and other distance costs between i and j; $\mathbf{a}$ = a constant; $\mathbf{f}$ = a decreasing function.

Assuming that there are **n** zones, the constraints are:

$$\sum_{i=1}^{n} F_{ij} = D_j \qquad (2)$$

and

$$\sum_{j=1}^{n} F_{ij} = S_i \qquad (3)$$

where $\mathbf{S_i}$= supply at i

A total transport expenditures constraint is usually applied as

$$\sum_{\mathbf{i}} \sum_{\mathbf{j}} \mathbf{F_{ij} \cdot c_{ij} = T} \qquad (4)$$

where **T** = total expenditures on transportation.

Replacing **a** in equation (1) by the multiplicative term $\mathbf{A_i \cdot B_j}$, where $\mathbf{A_i}$ and $\mathbf{B_j}$ are calculated so as to satisfy the constraints (2) and (3), the equation (1) may be transformed in the following production-attraction constrained model[1]:

$$\mathbf{F_{ij} = A_i B_j S_i D_j \cdot f\left(c_{ij}\right)} \qquad (5)$$

If all individual flows compatible with the overall constraints are equally probable, $\mathbf{f(c_{ij})}$ may be written as

$$\mathbf{F_{ij} = A_i B_j S_i D_j e^{-\beta c_{ij}}} \qquad (6)$$

where β = a parameter.

In this case the general transport-cost function is replaced by a negative exponential function ($\mathbf{e^{-\beta c_{ij}}}$). For high-weight - low value goods, when the cost-distance function may be more or less proportional, a linear, distance-decay function is recommended:

$$\mathbf{F_{ij} = A_i B_j S_i D_j e^{-\beta\gamma d_{ij}}} \qquad (7)$$

with $\mathbf{c_{ij}} = \gamma\ \mathbf{d_{ij}}$

Instead, for high-value goods or services a logarithmic distance function is usually employed: $c_{ij} = \gamma \ln d_{ij}$, that results in a power function:

$$\mathbf{F_{ij} = A_i B_j S_i D_j d_{ij}^{\;-\beta\gamma}} \qquad (8)$$

In conclusion, the choice between (7) and (8) depends upon the degree of distance decay. Where flows are rapidly attenuated by distance, the power function is preferred.

Gravity models for studying the migration flows. These models are based on the assumption that migration varies directly with the size of a force of attraction and inversely with distance. They require the force of attraction to be relevant, that is to show a connection with migration which may be considered reasonably constant and so able to be used in forecasting.

The simplest gravity model describes the population flows between two zones, i and j as $\mathbf{T_{ij} = G \cdot \frac{P_i \cdot P_j}{d_{ij}}}$, where $\mathbf{T_{ij}}$ = total population flows (in both directions) between i and j; $\mathbf{P_i}$, $\mathbf{P_j}$ = population of i, respectively j; **G** = a constant; $\mathbf{d_{ij}}$ = distance between i and j.

In order to determine the interaction between a given zone, i, and all other zones of the spatial system the following relation is applied:

$$\mathbf{T_{i1} + T_{i2} + \ldots + T_{ij} + \ldots + T_{in} = G_1 \frac{P_i \cdot P_1}{d_{i1}^{\;b1}} + G_2 \frac{P_i \cdot P_2}{d_{i2}^{\;b2}} + \ldots + G_j \frac{P_i \cdot P_j}{d_{ij}^{\;bj}} + \ldots + G_n \frac{P_i \cdot P_n}{d_{in}^{\;bn}}}$$

If the relative attraction of the competing zones is considered, the attraction between i and j is calculated as:

[1] According to Harry W. Richardson, *Regional and Urban Economics*, Pitman, 1973, p. 195:

$$\mathbf{A_i = 1 \Big/ \sum_{j=1}^{n} B_j \cdot D_j \cdot f\left(c_{ij}\right)} \quad \text{and} \quad \mathbf{B_j = 1 \Big/ \sum_{i=1}^{n} A_i \cdot S_i \cdot f\left(c_{ij}\right)}$$

$$\mathbf{T_{ij}} = \frac{\mathbf{G} \cdot \dfrac{\mathbf{P_i} \cdot \mathbf{P_j}}{\mathbf{d_{ij}}^{\mathbf{b}}}}{\mathbf{G_1}\dfrac{\mathbf{P_1}}{\mathbf{d_{i1}}^{\mathbf{b1}}} + \mathbf{G_2}\dfrac{\mathbf{P_2}}{\mathbf{d_{i2}}^{\mathbf{b2}}} + \ldots + \mathbf{G_k}\dfrac{\mathbf{P_k}}{\mathbf{d_{ik}}^{\mathbf{bk}}} + \ldots + \mathbf{G_n}\dfrac{\mathbf{P_n}}{\mathbf{d_{in}}^{\mathbf{bn}}}} = \frac{\mathbf{G} \cdot \mathbf{P_i} \cdot \mathbf{P_j} \cdot \mathbf{d_{ij}}^{-\mathbf{b}}}{\sum_{\substack{k=1 \\ k \neq i,j}}^{n} \mathbf{G_k} \cdot \mathbf{P_k} \cdot \mathbf{d_{ik}}^{-\mathbf{bk}}}$$

Assuming that the denominator is $\mathbf{A_i}$, then

$$\mathbf{T_{ij}} = \mathbf{G} \cdot \mathbf{P_i} \cdot \mathbf{A_i} \cdot \mathbf{P_j} \cdot \mathbf{d_{ij}}^{-\mathbf{b}}$$

Usually the model is written as

$$\mathbf{T_{ij}} = \mathbf{O_i} \cdot \mathbf{A_i} \cdot \mathbf{D_j} \cdot \mathbf{d_{ij}}^{-\mathbf{b}},$$

where $\mathbf{O_i} =$ number of trips originating in zone i; $\mathbf{D_j}$ = the attraction of zone j.

In general terms, the discussions on the nature of migration show that gravity models misattribute the role of distance, being likely that there is a threshold beyond which distance is an irrelevant consideration.

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