Equilibrium at the Producer's Level in the Measurable Uncertainty

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The papaer presents the essential aspects concerning the equilibrium at the producer's level in the measurable uncertainty. As a result, the selling price under uncertainty can be expressed as a function of the price under certainty, and thus is obtain the programm and the equation which lead to the solution, in all the situations presented.
***Keywords**: risk, measurable uncertainty, unmeasurable uncertainty, selling price.*

The dihotomy between risk and uncertainty was first presented by F.H. Knight in 1921, as it follows:

– *The risk* represents the *measurable uncertainty*, that is the evolution of a phenomenon is influenced by the probabilities for different states of the nature (in this case, the situation is known as *the risky situation*);

– *The unmeasurable uncertainty* represents the opposite case (the situation is known as the *uncertain situation*).

We will presents in the following a producer, for which we know:

- The quantity produced and offered by the producer, q;
- p_0 - the selling price under certainty environment;
- $C(q)$ - the variable cost;
- CF - the fix cost;
- Π - the payoff;

The payoff under the certainty environment, at the producer's level will be:

$$\Pi = p_0 q - C(q) - CF .$$

We will assume that the market price is uncertain and that its distribution is given by the lottery:

$\tilde{p} = L(\underline{p}, \overline{p}; \frac{1}{2}, \frac{1}{2})$, cu $\underline{p} < p_0 < \overline{p}$ si

$$E(\tilde{p}) = \frac{1}{2}(\underline{p}) + \frac{1}{2}(\overline{p}) = p_0$$

As a result, the selling price for the uncertainty case can be expressed as a function of the price under certainty:

$$\tilde{p} = p_0 + \tilde{y}, \text{ cu } \tilde{y} = L(\underline{p} + p_0, \overline{p} - p_0; \frac{1}{2}, \frac{1}{2})$$

si $E(\tilde{y}) = 0$ ($\tilde{y}$ is a normal lottery with fair probability)

The optimum quantity which a producer can offer results from solving the optimum problem:

$$[\max_q] EU(\tilde{\Pi} + w_0) = EU(\tilde{p}q - C(q) - CF + w_0),$$

where: $\tilde{\Pi}$ - is the random payoff; w_0 - is the initial endowment;

But: $EU(\tilde{p}q - C(q) - CF + w_0) =$

$$\frac{1}{2}U(\underline{p}q - C(q) - CF + w_0) + \frac{1}{2}U(\overline{p}q - C(q) - CF + w_0)$$

and so:

$$EU(\tilde{p}q - C(q) - CF + w_0) = \frac{1}{2}U(\underline{w}) + \frac{1}{2}U(\overline{w});$$

The first order condition leads to:

$$EU[(\tilde{p} - C'(q))U'(\tilde{p}q - C(q) - CF + W_0)] =$$

$$= \frac{1}{2}(\underline{p} - C'(q))U'(\underline{p}q - C(q) - CF + w_0) +$$

$$+ \frac{1}{2}(\overline{p} - C'(q))U'(\overline{p}q - C(q) - CF + w_0) = 0$$

From $U'(\bullet) > 0$ and $E(\bullet) = 0$ results that $\underline{p} \le C'(q) \le \overline{p}$.

We will make the following notation:

q^{**} - the optimum quantity produced by the firm under uncertainty, at the price $\tilde{p}$;

q^* - the optimum quantity produced by the firm under certainty at the price p_0;

$\underline{q}^*$ - the optimum quantity produced by the firm under certainty at the price $\underline{p}$;

$\overline{q}^*$ - the optimum quantity produced by the firm under certainty,
at the price $\overline{p}$;

As a result, we have the following table which presents the programm and the equation which leads to the solution, in any situation:

Table 1.

Price	Quantity	The program	The equation
p_0	q^*	$[\max]_q\{p_0 q - C(q) - CF\}$	$C'(q) = p_0$
$\tilde{p}$	q^{**}	$[\max]_q EU(\tilde{p}q - C(q) - CF + w_0)$	$\frac{1}{2}(\underline{p} - C'(q))U'(\underline{w}) + \frac{1}{2}(\overline{p} - C'(q))U'(\overline{w}) = 0$
$\underline{p}$	$\underline{q}^*$	$[\max]_q\{p_0 q - C(q) - CF\}$	$C'(q) = \underline{p}$
$\overline{p}$	$\overline{q}^*$	$[\max]_q\{p_0 q - C(q) - CF\}$	$C'(q) = \overline{p}$

Example 1. We consider a firm which can produce, from technical point of view, a maximum number of 150 units, for which we have.

$w_0 = 50.000$ u.m. (euro)

$CF = 3.000$ u.m. (euro)

$$C(q) = \begin{cases} 250q, \text{ daca } 0 \le q \le 70 \\ 400q\text{-}10500, \text{daca } 70 < q \le 150 \end{cases}$$

- the firm procedure is completly guided by the utility function $U(w) = \sqrt{w}$;
- under certainty, the firm will seel at the price $p_0 = 300$ euro;
- under uncertainty, the selling price will be $\underline{p} = 100$ euro and $\overline{p} = 500$ euro, with equal probabilities.

We look for the optimum quantity produced by the firm in the next situations:

a) there is no uncertainty on the price;

b) tehre are 2 bounded cases in the certain future, $\underline{p}$ and $\overline{p}$;

c) there is uncertainty on the price.

Solution:

a) From the optimum problem:

$$[\max]_q \Pi = p_0 q - C(q) - CF$$

applying the optimum necessary condition we get $\frac{d\Pi}{dq} = 0$ which leads to

$$p_0 - C'(q) = 300 - C'(q) =$$

$$= 300 - \begin{cases} 250, \text{ daca } 0 \le q \le 70 \\ 400, \text{ daca } 70 < q \le 150 \end{cases}$$

$$= \begin{cases} 50, \text{ daca } 0 \le q \le 70 \\ \text{-}100, \text{ daca } 70 < q \le 150 \end{cases}$$

Observation: For a quantity between 0 and 70 units, the marginal payoff is positive (50 euro for each unit), which leads to a growth in the obtained quantity till the 70 units, because one unit extra (the 71 unit) will lead to a 100 euro loss. Consequenty, the optimum quantity will be of 70 units, $q^* = 70$, which leads to a payoff of $70 \times 300 - 70 \times 250 - 3000 = 500$ euro.

b) We study each case:

- *For* $\underline{p} = 100$ *euro*, the optimum problem is:

$$[\max]_q \Pi = \underline{p}q - C(q) - CF$$

The necessary optimum condition is:

$$\frac{d\Pi}{dq} = 0,$$

$$\underline{p} - C'(q) = 100 - \begin{cases} 250, \text{ daca } 0 \le q \le 70 \\ 400, \text{ daca } 70 < q \le 150 \end{cases}$$

$$= \begin{cases} -150, \text{ daca } 0 \le q \le 70 \\ \text{-}300, \text{ daca } 70 < q \le 150 \end{cases}$$

In this case, as we can see, every unit of goods, if would be produced, would automaticly lead to losses.

So, the firm will take the decision not to produce: $q^* = 0$, and the payoff will actually be a loss of 3000 euro, beacuse of the fix cost.

- for $\overline{p} = 500$ euro we'll have, as before:

$$[\max]_q \Pi = \overline{p}q - C(q) - CF$$

CNO: $\frac{d\Pi}{dq}=0$, this leads to

$$\overline{p}-C'(q)=500-\begin{cases}250, \text{daca } 0\le q\le 70\\ 400, \text{ daca } 70<q\le 150\end{cases}$$
$$=\begin{cases}250, \text{daca } 0\le q\le 70\\ 100, \text{ daca } 70<q\le 150\end{cases}$$

We can see that the marginal payoff is positive on the both branches, so we can get $\overline{q}^*=150$ units (each extra unit produced till 150 will bring a payoff).

The payoff will be of: 150*500-400*150+ +10500-3000=22500 euro.

c) The optimum problem in this case is:

$$[\max_q]\ EU(\tilde{\Pi}+w_0)=EU(\tilde{p}q-C(q)-CF+w_0)=$$
$$=\frac{1}{2}U(\underline{p}q-C(q)-CF+w_0)+\frac{1}{2}U(\overline{p}q-C(q)-CF+w_0)=$$
$$=\frac{1}{2}\sqrt{100q-C(q)+47000}+\frac{1}{2}\sqrt{500q-C(q)+47000}$$

Aplying the first order condition we get:

$$\frac{1}{2}\cdot\left(\frac{100-C'(q)}{2\sqrt{100q-C(q)+47000}}\right)+\frac{1}{2}\left(\frac{500-C'(q)}{2\sqrt{500q-C(q)+47000}}\right)=0$$

or:

$$\begin{cases}\frac{1}{2}\left(\frac{100-250}{2\sqrt{100q-250q+47000}}\right)+\frac{1}{2}\left(\frac{500-250}{2\sqrt{500q-250q+47000}}\right)=0, \text{daca } 0\le q\le 70\\ \frac{1}{2}\left(\frac{100-400}{2\sqrt{100q-400q+57500}}\right)+\frac{1}{2}\left(\frac{500-400}{2\sqrt{500q-400q+57500}}\right)=0, \text{ daca } 70<q\le 150\end{cases}$$

$$\Leftrightarrow\begin{cases}\frac{75}{2\sqrt{47000-150q}}=\frac{125}{2\sqrt{47000+250q}}, \text{ daca } 0\le q\le 70\\ \frac{75}{\sqrt{57500-300q}}=\frac{25}{\sqrt{57500+100q}}, \text{ daca } 70<q\le 150\end{cases}$$

$$\Leftrightarrow\begin{cases}\frac{3}{\sqrt{47000-150q}}=\frac{5}{\sqrt{47000+250q}}, \text{ daca } 0\le q\le 70\\ \frac{3}{\sqrt{57500-300q}}=\frac{1}{\sqrt{57500+100q}}, \text{ daca } 70<q\le 150\end{cases}$$

$$\Leftrightarrow\begin{cases}9(47000+250q)=25(4700-150q), \text{ daca } 0\le q\le 70\\ 9(57500+100q)=57500-300q, \text{ daca } 70<q\le 150\end{cases}$$

$$\Leftrightarrow\begin{cases}q\cong 125.3, \text{ daca } 0\le q\le 70\\ q\cong -383.3, \text{ daca } 70<q\le 150\end{cases}$$

both branches being impossible.

As a conclusion, the firm can not offer a production under the conditions form the c), meaning that: $q^{**}=0$; this implies that the random payoff will actually be a loss of 3000 euro with the probability of $\frac{1}{2}$ both at the price $\underline{p}$, as at the price $\overline{p}$.

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