Some observations on a particular type of k-order nonlinear discrete determinist exchange rate models

Prof. Stefan BALINT, PhD.
Faculty of Mathematics, West University Timisoara
Lect. Mirela-Catrinel VOICU, PhD.
Faculty of Economic Sciences, West University Timisoara

In this work we present some qualitative results for a particular type of k-order exchange rate models. These results concern the existence of the fixed point, it's stability and it's attraction domain and the existence of the period-two cycles. Given the nonlinear nature, these systems can display a more complex evolution (chaotic behavior, period doubling-bifurcation, limit cycle and period-p cycle with p>2). For k=2 and k=3 we have presented in [4]-[7] mathematical and numerical results. The present work objective is to generalize the results obtained for the second-order dynamical system and the third-order dynamical system. The algorithms implementation must to be make for more value of k and this is practical very difficult (specially for chaotic behavior study) and in the numerical simulations the initial conditions values and parameters are constants. For this reason, now, we present only mathematical results. However, our study conduces to interesting similarities between the dynamics of this type models.
Keywords: k-order nonlinear discrete determinist exchange rate models

1 Introduction A basic equation generally used to model the exchange rate is the following (see [1], [2]):

(1) $S_t = X_t E_t(S_{t+1})^b$

where S_t is the exchange rate at time *t*; X_t can be thought of as a reduced form equation describing the structure of the model and the exogenous variables that drives the exchange rate in the period *t*; $E_t(S_{t+1})$ is the expectation held today (time *t*) in the market about next period *t+1* exchange rate; b is the discount factor that speculators use to discount the future expected exchange rate (*0<b<1*).

This model permits to take into account two components for forecasting, a forecast made by the chartists and a forecast made by the fundamentalists:

$$(2) \qquad E_t(S_{t+1})/S_{t-1} = (E_{ct}(S_{t+1})/S_{t-1})^{m_t}(E_{ft}(S_{t+1})/S_{t-1})^{1-m_t}$$

where $E_t(S_{t+1})$ has the same meaning as before; $E_{ct}(S_{t+1})$ and $E_{ft}(S_{t+1})$ are respectively the forecasts made by the chartists and the fundamentalists; m_t is the weight given by the chartists and $1-m_t$ is the weight given by the fundamentalists at time *t*.

Te fundamentalists are assumed to calculate the equilibrium exchange rate S_t^*. The steady state can be calculated by imposing $E_{ft}(S_{t+1}) = S_t = S_{t-1}$. This implies that $S_t^* = (X_t)^{1/(-b)}$.

When the current exchange rate is above/below relative to the equilibrium rate, the fundamentalists expect that the future exchange rate to go down/increase. The fundamentalists expect the market rate to return to that fundamental rate S_t^* with the speed a during the next period, if they observe a deviation today, then their forecasts are obtained from the following model:

(3) $$\frac{E_{ft}(S_{t+1})}{S_{t-1}} = \left(\frac{S^*_{t-1}}{S_{t-1}}\right)^a, a > 0.$$

The chartist use the past of the exchange rate to detect patterns that they extrapolate in the future. An equation, which give a very general description of different models used by chartists, is the following:

(4) $$\frac{E_{ct}(S_{t+1})}{S_{t-1}} = f(S_{t-1},...,S_{t-N}).$$

It is possible to specify such a rule in very general terms as follows:

(5) $$\frac{E_{ct}(S_{t+1})}{S_{t-1}} = \left(\frac{S_{t-1}}{S_{t-2}}\right)^{c_1}\left(\frac{S_{t-2}}{S_{t-3}}\right)^{c_2}...\left(\frac{S_{t-N+1}}{S_{t-N}}\right)^{c_{N-1}}$$

The weight m_t, in (2), given by chartists is

(6) $$m_t = \frac{1}{1+b(S_{t-1}-S^*_{t-1})^2}, \quad b > 0.$$

The parameter b measures the precision degree of the fundamentalists' estimates.

In this work, for the chartists' behavior one assumed the following models:

(7) $$\frac{E_{ct}(S_{t+1})}{S_{t-1}} = \left(\frac{S_{t-1}}{S_{t-k}}\right)^c, \quad c > 1, \quad k \geq 2, \quad k \in N.$$

For the sake of simplicity, the exogenous variable X_t are considered equal to one.

Using these models for the chartists, in the equation (1), we obtain ***k***-order discrete dynamical systems.

Now, substituting the equation (3) and (7) into (2), and (2) into (1), we get the following system:

(8) $$S_t = S_{t-1}^{[(c+a)m_t b+(1-a)b]} S_{t-k}^{-cm_t b}.$$

If we denote $s_t = \ln S_t$, then the system (8) can be rewrite in the following way:

(9) $$s_t = \left(\frac{(c+a)b}{1+b(e^{s_{t-1}}-1)^2} + (1-a)b\right)s_{t-1} - \frac{cb}{1+b(e^{s_{t-1}}-1)^2}s_{t-k}$$

with $s_t \in R, \quad \forall t \in Z$. We also can rewrite (9) in a vectorial form and we get the expression:

(10) $$(s_{t+2},...,s_{t+k+1}) = F(s_{t+1},...,s_{t+k})$$

if we define the function $F: R^k \to R^k$, such that

$$F(x_1,...,x_k) = (F_1(x_1,...,x_k),...,F_k(x_1,...,x_k))$$

where $F_i(x_1,...,x_k) = x_{i+1}, \quad i = \overline{1,k-1}$ and $F_k(x_1,...,x_k) = j(x_k x_k + f(x_k)x_1$ and

$$j(x) = \frac{(c+a)b}{1+b(e^x-1)^2} + (1-a)b \text{ and}$$

$$f(x) = -\frac{cb}{1+b(e^x-1)^2}.$$

2. Fixed point for the system (10). Existence and unicity, stability and attraction domain

2.1. Fixed point – existence and unicity

Proposition 1. In the case in which $c > 1$ $b \in (0,1)$, $a > 0$ and $b > 0$, the system (10) has only the fixed point $(0,...,0)$ ($(0,...,0) \in R^k$).

Observation 1. (The fixed point stability). The Jacobian matrix of F at $(0,...,0)$ is the matrix

$$J = \begin{pmatrix} 0 & 1 & ... & 0 \\ ... & & & \\ 0 & 0 & ... & 1 \\ -cb & 0 & ... & (c+1)b \end{pmatrix}$$

and this matrix has the determinant

$$(-1)^k l^k + (-1)^{k-1}(1+c)b\, l^{k-1} + (-1)^k bc.$$

Calculating the eigenvalues of the Jacobian matrix we can establish when the fixed point is stable or unstable.

We give not a mathematical solution for this problem, but we make the following observation:

Now, we fix *c=2*. Using computational results, we obtain that: 1) if *k=2* then for $b \in (0,0.5)$ the fixed point $(0,0)$ is stable and for $b \in (0.5,1)$ the fixed point is unstable; 2) if *k=3* then for $b \in (0, 0.3165)$ the fixed point $(0,0,0)$ is stable and for $b \in (0.3165, 1)$ the fixed point is unstable;

3) if *k=4* then for $b \in (0, 0.2654)$ the fixed point (0,0,0,0) is stable and for $b \in (0.2654, 1)$ the fixed point is unstable;
4) if *k=5* then for $b \in (0, 0.2428)$ the fixed point (0,0,0,0,0) is stable and for $b \in (0.2428, 1)$ the fixed point is unstable;
5) if *k=6* then for $b \in (0, 0.2306)$ the fixed point (0,0,0,0,0) is stable and for $b \in (0.2306, 1)$ the fixed point is unstable;
6) if *k=10* then for $b \in (0, 0.2119)$ the fixed point (0,0,0,0,0,0,0,0,0,0) is stable and for $b \in (0.2119, 1)$ the fixed point is unstable. However, from these empirical results, we can observe difference of dynamics, between different order systems.

2.2 Attraction domain for the fixed point

In the case in which a fixed point is stable, it is important to study it's attraction domain. In order to make this study, now, we give the following results:

Proposition 2. Under the assumption: $c > 1$, $b \in \left(0, \dfrac{1}{2c+1}\right]$, $a \in (0,1)$, , $b > 0$, $s_t \in R$, $t \in Z$, the following relations are verified:

c.1. For $s_{t+1}, s_{t+k} < 0$ **:**

c.1.1. if $s_{t+1} \in \left(-\infty, \dfrac{(c+1)+(1-a)b(e^{s_{t+1k}}-1)^2}{c} s_{t+k}\right)$ then $-s_{t+1} > s_{t+k+1} > 0 > s_{t+k} > s_{t+1}$

c.1.2. if $s_{t+1} = \dfrac{(c+1)+(1-a)b(e^{s_{t+k}}-1)^2}{c} s_{t+k}$ then $s_{t+k+1} = 0 > s_{t+k} > s_{t+1}$

c.1.3. if $s_{t+1} \in \left(\dfrac{(c+1)+(1-a)b(e^{s_{t+k}}-1)^2}{c} s_{t+k}, \dfrac{(c+1)b+(1-a)bb(e^{s_{t+k}}-1)^2}{1+cb+b(e^{s_{t+k}}-1)^2} s_{t+k}\right)$ then $0 > s_{t+k+1} > s_{t+k}$, $0 > s_{t+k+1} > s_{t+1}$

c.1.4. if $s_{t+1} = \dfrac{(c+1)b+(1-a)bb(e^{s_{t+k}}-1)^2}{1+cb+b(e^{s_{t+k}}-1)^2} s_{t+k}$ then $0 > s_{t+k+1} = s_{t+1} > s_{t+k}$

c.1.5. if $s_{t+1} \in \left(\dfrac{(c+1)b+(1-a)bb(e^{s_{t+k}}-1)^2}{1+cb+b(e^{s_{t+k}}-1)^2} s_{t+k}, 0\right)$ then $0 > s_{t+1} > s_{t+k+1} > s_{t+k}$

c.2. For $s_{t+1}, s_{t+k} > 0$ **:**

c.2.1. if $s_{t+1} \in \left(0, \dfrac{(c+1)b+(1-a)bb(e^{s_{t+k}}-1)^2}{1+cb+b(e^{s_{t+k}}-1)^2} s_{t+k}\right)$ then $s_{t+k} > s_{t+k+1} > s_{t+1} > 0$

c.2.2. if $s_{t+1} = \dfrac{(c+1)b+(1-a)bb(e^{s_{t+k}}-1)^2}{1+cb+b(e^{s_{t+k}}-1)^2} s_{t+k}$ then $s_{t+k} > s_{t+k+1} = s_{t+1} > 0$

c.2.3. if $s_{t+1} \in \left(\dfrac{(c+1)b+(1-a)bb(e^{s_{t+k}}-1)^2}{1+cb+b(e^{s_{t+k}}-1)^2} s_{t+k}, \dfrac{(c+1)+(1-a)b(e^{s_{t+k}}-1)^2}{c} s_{t+k}\right)$ then $s_{t+k} > s_{t+k+1} > 0$, $s_{t+1} > s_{t+k+1} > 0$

c.2.4. if $s_{t+1} = \dfrac{(c+1)+(1-a)b(e^{s_{t+k}}-1)^2}{c} s_{t+k}$ then $s_{t+1} > s_{t+k} > s_{t+k+1} = 0$

c.2.5. if $s_{t+1} \in \left(\dfrac{(c+1)+(1-a)b(e^{s_{t+k}}-1)^2}{c} s_{t+k}, \infty\right)$ then $s_{t+1} > s_{t+k} > 0 > s_{t+k+1} > -s_{t+1}$

c.3. For $s_{t+1} < 0, s_{t+k} > 0$ **:**

c.3.1. if $s_{t+1} \in \left(-\infty, \frac{(c+1)b-1+[(1-a)b-1]b(e^{s_{t+k}}-1)^2}{cb} s_{t+k}\right)$ then

$-s_{t+1} > s_{t+k+1} > s_{t+k} > 0 > s_{t+1}$

c.3.2. if $s_{t+1} = \frac{(c+1)b-1+[(1-a)b-1]b(e^{s_{t+k}}-1)^2}{cb} s_{t+k}$ then $s_{t+k+1} = s_{t+k} > 0 > s_{t+1}$

c.3.3. if $s_{t+1} \in \left(\frac{(c+1)b-1+[(1-a)b-1]b(e^{s_{t+k}}-1)^2}{cb} s_{t+k}, 0\right)$ then $s_{t+k} > s_{t+k+1} > 0 > s_{t+1}$

c.4. For $s_{t+1} > 0, s_{t+k} < 0$ **:**

c.4.1. if $s_{t+1} \in \left(0, \frac{(c+1)b-1+[(1-a)b-1]b(e^{s_{t+k}}-1)^2}{cb} s_{t+k}\right)$ then $s_{t+1} > 0 > s_{t+k+1} > s_{t+k}$

c.4.2. if $s_{t+1} = \frac{(c+1)b-1+[(1-a)b-1]b(e^{s_{t+k}}-1)^2}{cb} s_{t+k}$ then $s_{t+1} > 0 > s_{t+k+1} = s_{t+k}$

c.4.3. if $s_{t+1} \in \left(\frac{(c+1)b-1+[(1-a)b-1]b(e^{s_{t+k}}-1)^2}{cb} s_{t+k}, \infty\right)$ then

$s_{t+1} > 0 > s_{t+k} > s_{t+k+1} > -s_{t+1}$

c.5. if $s_{t+1} < 0$ and $s_{t+k} = 0$ then $0 < s_{t+k+1} < -s_{t+1}$

c.6. if $s_{t+1} > 0$ and then $-s_{t+1} < s_{t+k+1} < 0$

c.7. if $s_{t+1} = 0$ and $s_{t+k} < 0$ then $s_{t+k} < s_{t+k+1} < 0$

c.8. if $s_{t+1} = 0$ et $s_{t+k} > 0$ then $0 < s_{t+k+1} < s_{t+k}$

Remark 1. For $c > 1, a \in (0,1),\ b > 0$ and $b \in \left(0, \frac{1}{2c+1}\right]$ we find that $|s_{t+k+1}| \leq \max\{|s_{t+1}|, |s_{t+k}|\},\ \forall s_t \in R, t \in Z$. This relation implies that

$$\max\{|s_{t+jk+1}|,...,|s_{t+(j+1)k}|\} \leq \max\{|s_{t+(j-1)k+1}|,...,|s_{t+jk}|\}, \forall t \in Z, j \in Z$$

If we define: $s^*_{t+jk} = s_{t+(j-1)k+i}$ if $|s_{t+(j-1)k+i}| = \max\{|s_{t+(j-1)k+1}|,...,|s_{t+jk}|\}$, where $j \in Z$ $i = \overline{1,k}$, then the sequence $\{|s^*_{t+jk}|\}_{t\in Z}$ is monotonously decreasing and positive and this implies that the sequence $\{|s^*_{t+jk}|\}_{t\in Z}$ is convergent. If $p = \lim_{t\to\infty} |s^*_{t+jk}|$, then

1) $\lim_{t\to\infty} s^*_{t+jk} = p$ or 2) $\lim_{t\to\infty} s^*_{t+jk} = -p$ or

3) $\lim_{t\to\infty} s^*_{t+jk} = p$ for $t + jk \in T$, $\lim_{t\to\infty} s^*_{t+jk} = -p$ for $t + jk \in T'$, where $T, T' \subset Z$ with $T \cap T' = \varnothing$, $T \cup T' = Z$ and T, T' are infinite.

Using the Proposition 2 and the Remark 1, we give the following result:

Proposition 3. For $c > 1, a \in (0,1),\ b > 0$ and $b \in \left(0, \frac{1}{2c+1}\right]$ the limit p is null for any initial condition of the system (10). This implies that the fixed point $(0,...,0)$ $((0,...,0) \in R^k)$ is globally attractive.

From the Proposition 3 and Observation 1 we get the following results:

Proposition 4. The fixed point $(0,...,0)$ $((0,...,0) \in R^k)$ is stable for $b \in (0, y(c,k))$, where $\frac{1}{2c+1} \leq y(c,k) < 1$.

3. Period-two cycles for the system (10)

A period-2 point of the system (10) is a solution of the equation $(s_1,...,s_k) = F^2(s_1,...,s_k)$ where $(s_1,...,s_k) \neq F(s_1,...,s_k)$. The relations $(s_2,...,s_{k+1}) = F(s_1,...,s_k)$, $(s_3,...,s_{k+2}) = F(s_2,...,s_{k+1})$ and $(s_3,...,s_{k+2}) = (s_1,...,s_k)$ imply that

1) $(s_2, s_1,..., s_1, s_2) = F(s_1, s_2,..., s_2, s_1)$ and $(s_1, s_2,..., s_2, s_1) = F(s_2, s_1,..., s_1, s_2)$ if k is an uneven number

2) $(s_2, s_1,..., s_2, s_1) = F(s_1, s_2,..., s_1, s_2)$ and $(s_1, s_2,..., s_1, s_2) = F(s_2, s_1,..., s_2, s_1)$ if k is an even number

where the vectors are in R^k.

3.1 *k* is an even number

Proposition 5. Under the assumption $c > 1$ and $b \in (0,1)$, if $a \in \left(0, \quad 1+\frac{1}{b}\right]$ or $a \in \left(1+\frac{1}{b}, \infty\right)$ and $b \in \left(0, \frac{(c+1)b^2(a-1)+1+cb}{(a-1)^2b^2-1}\right]$ the system (10) has no cycle of period two.

If $a \in \left(1+\frac{1}{b}, \infty\right)$ and

$$b \in \left(\frac{(c+1)b^2(a-1)+1+cb}{[(a-1)^2b^2-1]}, \infty\right)$$

the system (10) has only a cycle of period two. This cycle is $\{(s_1, s_2,..., s_1, s_2), (s_2, s_1,..., s_2, s_1)\}$ where s_1 and s_2 are the solutions of the equation

$$\frac{j\left(\frac{j(x)}{1-f(x)}x\right)}{1-f\left(\frac{j(x)}{1-f(x)}x\right)}\frac{j(x)}{1-f(x)} = 1,$$

which means that s_1 and s_2 are the solutions of the equation

$$\frac{(c+1)b+(1-a)bb\left(e^{\frac{(c+)b+(1-a)bb(e^x-1)^2}{1+cb+b(e^x-1)^2}x}-1\right)^2}{1+cb+b\left(e^{\frac{(c+)b+(1-a)bb(e^x-1)^2}{1+cb+b(e^x-1)^2}x}-1\right)^2}\frac{(c+1)b+(1-a)bb(e^x-1)^2}{1+cb+b(e^x-1)^2} = 1.$$

The numbers s_1 and s_2 verify the relation $s_1 s_2 < 0$. Let $s_1 > 0$ to be the positive number.

If $b \in \left(\frac{(c+1)b^2(a-1)+1+cb}{[(a-1)^2b^2-1]}, \frac{(1+2c)b+1}{[(a-1)b-1]}\right)$ then $s_1 > \ln\left(1+\sqrt{\frac{1}{b}\frac{1+(2c+1)b}{[(a-1)b-1]}}\right)$ and $s_2 < -\ln\left(1+\sqrt{\frac{1}{b}\frac{1+(1+2c)b}{[(a-1b-1)]}}\right)$.

If $b \in \left[\frac{(2c+1)b+1}{[(a-1)b-1]}, \infty\right)$ then $s_1 \in \left(\ln\left(1+\sqrt{\frac{1}{b}\frac{(2c+1)b+1}{[(a-1)b-1]}}\right), -\ln\left(1-\sqrt{\frac{1}{b}\frac{(2c+1)b+1}{[(a-1)b-1]}}\right)\right)$ and

$$s_2 \in \left(\ln\left(1-\sqrt{\frac{1}{b}\frac{(2c+1)b+1}{[(a-1)b-1]}}\right), -\ln\left(1+\sqrt{\frac{1}{b}\frac{(2c+1)b+1}{[(a-1)b-1]}}\right)\right).$$

From the Propositions 4 and 5 we obtain the following result:

Proposition 6. If $c > 1$, $b \in \left(0, \frac{1}{2c+1}\right)$, $a \in \left(1+\frac{1}{b}, \infty\right)$ and

$b \in \left(\frac{(c+1)b^2(a-1)+1+cb}{[(a-1)^2 b^2 - 1]}, \infty \right)$, then the fixed point $(0,...,0)$ ($(0,...,0) \in R^k$) is only locally stable.

Observation 2. We can observe that for the period-two cycle existence, in this case, it is not important k (the order of the system).

3.2 *k* is an uneven number

Proposition 7. Under the assumption $c > 1$ and $b \in (0,1)$, if $a \in \left(0, \quad 1+\frac{1}{b}\right]$ or $a \in \left(1+\frac{1}{b}, \infty\right)$ and $b \in \left(0, \frac{b^2(a-1)+1}{(a-1)^2 b^2 - 1}\right]$ the system (10) has no cycle of period two.

If $a \in \left(1+\frac{1}{b}, \infty\right)$ and $b \in \left(\frac{b^2(a-1)+1}{[(a-1)^2 b^2 - 1]}, \infty\right)$ the system (10) has only a cycle of period two. This cycle is $\{(s_1, s_2, ..., s_2, s_1), (s_2, s_1, ..., s_1, s_2)\}$ where s_1 and s_2 are the solutions of the equation

$(j((j(x)+f(x))x)+f((j(x)+f(x))x))(j(x)+f(x)) = 1$, which means that s_1 and s_2 are the solutions of the equation

$$\left(\frac{ab}{1+b\left(e^{\left(\frac{ab}{1+b(e^x-1)^2}+(1-ab)\right)x}-1\right)^2} + (1-a)b \right) \left(\frac{ab}{1+b(e^x-1)^2} + (1-ab) \right) = 1 .$$

The numbers s_1 and s_2 verify the relation $s_1 s_2 < 0$. Let $s_1 > 0$ to be the positive number.

If $b \in \left(\frac{b^2(a-1)+1}{[(a-1)^2 b^2 - 1]}, \frac{b+1}{[(a-1)b-1]} \right)$ then

$$s_1 > \ln\left(1+\sqrt{\frac{1}{b}\frac{1+b}{[(a-1)b-1]}}\right) \text{ and } s_2 < -\ln\left(1+\sqrt{\frac{1}{b}\frac{1+b}{[(a-1)b-1]}}\right).$$

If $b \in \left[\frac{b+1}{[(a-1)b-1]}, \infty \right)$ then

$$s_1 \in \left(\ln\left(1+\sqrt{\frac{1}{b}\frac{b+1}{[(a-1)b-1]}}\right), -\ln\left(1-\sqrt{\frac{1}{b}\frac{b+1}{[(a-1)b-1]}}\right) \right) \text{ and}$$

$$s_2 \in \left(\ln\left(1-\sqrt{\frac{1}{b}\frac{b+1}{[(a-1)b-1]}}\right), -\ln\left(1+\sqrt{\frac{1}{b}\frac{b+1}{[(a-1)b-1]}}\right) \right)$$

From the Propositions 4 and 7 we obtain the following result:

Proposition 8. If $c > 1$, $b \in \left(0, \frac{1}{2c+1}\right)$, $a \in \left(1+\frac{1}{b}, \infty\right)$ and $b \in \left(\frac{b^2(a-1)+1}{[(a-1)^2 b^2 - 1]}, \infty\right)$, then the fixed point $(0,...,0)$ ($(0,...,0) \in R^k$) is only locally stable.

Observation 3. We can observe that for the period-two cycle existence, in this case, it is not important k (the order of the system) and it is not important c.

Conclusion

This study conduce at conclusion that there it exists similarities between the dynamics of the systems studied. These results are interesting from a mathematical viewpoint. But also, these results conduce at economical interpretations.

References

1. De Grauwe P. (1996) - *International money: Postward trends and Theories,* Oxford University Press

2. De Grauwe P., Dewachter H., Embrechts M. (1993) - *Exchange rate theory: chaotic models of foreign exchange markets*, Blackwell Publishers

3. Strogatz S.H. (1994) - *Nonlinear dynamics and Chaos, Studies in nonlinearity,* Addison-Wesley.

4. D. Guégan, M.C.Voicu – *Chaotic behavior in a macroeconomic model*, Université de Reims, UMR 6056, 2000, France

5. D. Guégan, St. Balint, M.C. Voicu - *Stabilité asymptotique et domaine d'attraction du point d'équilibre dans le cas d'un système dynamique qui décrit l'évolution du taux de change* - 5éme Colloque Franco-Roumain de Mathématique Appliquées, Constanta, 28 août –1 er septembre 2000

6. M.C.Voicu, C. Chilarescu, E. Cazan, St.Balint - *On the dynamical behavior of the exchange rate in a nonlinear model*, XV Reunión de Asepelt**,** La Facultad de Ciencias Económicas y Empresariales de la Universidad de A Coruña, 21 y 22 de junio de 2001, Spania

7. D. Guégan, St. Balint, M.C. Voicu - *Une étude des types de comportements dynamiques, qui peuvent être rencontrés dans modèles d'évolutions du taux de change avec anticipations formulées avec chartistes et fondamentalistes*- 6ème Colloque Franco-Roumain de Mathématiques Appliquées du 2 au 6 septembre 2002, Perpignan (France)