

Applications of credibility theory - The first part -

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Abstract: *In this paper, we will discuss some practical insurance problems that can be solved by means of credibility theory. All numerical results in this paper were obtained using the software package CRAC 2.0, which uses Jewell's hierarchical model. In Section 1 a brief overview of the structure and the possibilities of the program is given. It will be shown sectors can be determined in order to use the hierarchical model that is built into the software. Furthermore a general structure for defining insurance problems to be solved by CRAC 2.0 will be discussed. In Section 2 we will combine all elements from the previous subsections (introduction and philosophy; subdivision of the portfolio into sectors; definition of the problem; a standard procedure; structure of CRAC/ input) into one example.*

The goal of this is to illustrate how practical problems can be translated into problems that can be handled by the CRAC – software. In the following articles – the second and the third – we will concentrate on the results obtained rather than on the practical problems arising from working with this software package.

Key words: *hierarchical scheme, sector, contract, credibility results, the risk premium.*

1 Structure of the software

The CRAC 2.0 input is designed in such a way that quite a number of practical problems can be solved. We describe the input format for the different variables. An example will illustrate the general guidelines.

The CRAC – software is based on a two – level semi – linear hierarchical credibility model. It is semi – linear in the sense that some (fixed) transformation of the claim figures is used. It is two-level hierarchical, as the individual contracts on the down-level are grouped together in sectors on an intermediate level. So decisions can be made on three different levels:

- the top-level based on collective estimators for the whole portfolio;
- the in-between sector level based on the estimators for the different sectors defined by the user;
- the down-level based on the individual estimators for all contracts.

For every contract in the portfolio, data for T observation periods can be given (a contract can also be an average of a group of contracts). For a successful application of the software, T should be at least 3. The

portfolio is split into a number of sectors and each of these sectors contains individual contracts. In this way we obtain three decision levels: global (for the portfolio as a whole), denoted by m , sectorial, denoted by N_p^a and individual, denoted by M_{pj}^a .

The sectors can be determined by CRAC 2.0 according to either one, two or three different characteristics. Each individual record contains up to 3 different variables ranging from 1 to S1, S2 and S3 respectively. Then the following subdivisions are possible:

- on the basis of one of these variables resulting in S1, S2 or S3 sectors;
- on the basis of two combined criteria resulting in two subsequent subdivisions in cells according to the chosen pairs of S – variables. This gives for instance a total of $S1 \cdot S2$ cells if the first two criteria are used;
- on the basis of all three variables, producing $S1 \cdot S2 \cdot S3$ sectors.

Consider the two following examples:

Example 1: Individual fire insurance

A subdivision into sectors might be based on the following criteria:

- owned house or rented house;
- size of the house: small, medium or big house; (SH, MH, BH);
- geographical position of the house (district, state, ...).

The choice of this subdivision may be brought on by statistical differences of the various groups, or simply imposed by the management.

Example 2: Health insurance

We consider the principle of this type of insurance to be that a firm insures all its employees as a group for medical costs. So the client of the insurance company is the firm and not the individual employee.

A subdivision of a portfolio of such contracts might be based on the following criteria:

- kind of activities of the firm: services, chemical, agricultural, ...;
- geographical position of the firm: e.g. Northern, Central and Southern part of the country;
- turnover of the firm: a number of classes could be defined.

The total portfolio in these two examples can then be split up according to one, two or all three of these criteria. It is obvious that not only the sectors but also the quantity to be calculated will be different from one application to another. In fire insurance we normally will be interested in calculating a permillage of the capital insured, whereas in the problem of health insurance we want to calculate a premium (in some currency) per individual insured. For this reason a standard procedure is provided within the CRAC – software. The standard form for defining problems is as follows:

$$X_{pjr} = \frac{((\text{numerator})_{pjr} - \text{deductible})_+}{\text{factor} \cdot (\text{denominator})_{pjr}} \quad (1),$$

where $q_+ = \max\{0, q\}$, p denotes the sector this policy belongs to, j denotes the contract and r denotes the observation period. For the numerator we usually take claim payments, while the denominator expresses some measure of exposure like premium volume or number of partici-

pants. We will illustrate this standard form for the two examples mentioned previously

Example 1: Individual fire insurance

When we are interested in calculating a risk premium, as a permillage of the capital insured for each sector as defined earlier, the numerator is the amount paid by the insurance company, the denominator is the capital insured. By choosing the scaling factor as 0,001 the quantity X_{pjr} represents the risk permillage we are interested in.

Example 2: Health insurance

In this example we calculate a risk premium per individual, so we have to take the numerator as the amount paid by the insurer; the denominator represents the number of insured employees in the firm.

An input – file to be used in CRAC 2.0 must contain the following data for each record:

1. Policy number or identification number.
2. Specification of the sector. Three fields, containing the values according to which the subdivisions in sectors is organized. These variables range from 1 to S1, S2 or S3.
3. Number of observations. This value must be greater than zero and at most equal to T.
4. For each observation period i ($i = -1, -2, \dots, -T$) the following fields:
 - An indicator which is set to zero in case no observation for the period i is available, otherwise it is 1;
 - A numerator for the observation period i ;
 - A denominator for the observation period i ; this field can only contain a value of zero when field 4a indicates a non-available period;
 - A weight factor for the observation period i ; reflects the importance in the portfolio of each individual contract.

2. A practical example

To determine the risk premium for individual fire insurance, assume statistical information is available for the last four years. The risk premium must be expressed as a permillage of the capital insured. The user

judges a subdivision into the following sections necessary:

- 1) houses in the North (N) and houses in the South (S);
- 2) houses with an insured capital of less than \$ 100.000 (SH), from \$ 250.000 (MH) and more than \$ 250.000 (BH);
- 3) houses in the city (IC) and houses outside the city (OC).

It is not the intention to set up an individual tariff. The only aim of the user is to set up a sectorial investigation. The user should make the following selections:

1. $T = 4$;
2. no individual calculations;
3. $S1 = 2$ (N/S), $S2 = 3$ (SH/MH/BH) and $S3 = 2$ (IC/OC);
4. the variables defining the subdivision in sections; when the three criteria are combined, the following sectors appear ($S1 \cdot S2 \cdot S3 = 2 \cdot 3 \cdot 2 = 12$ sectors):

- | | | | |
|---------------|----|----|----------------|
| - sector 1: N | SH | IC | - sector 7: S |
| | SH | IC | |
| - sector 2: N | SH | OC | - sector 8: S |
| | SH | OC | |
| - sector 3: N | MH | IC | - sector 9: S |
| | MH | IC | |
| - sector 4: N | MH | OC | - sector 10: S |
| | MH | OC | |
| - sector 5: N | BH | IC | - sector 11: S |
| | BH | IC | |

- | | | | |
|---------------|----|----|----------------|
| - sector 6: N | BH | OC | - sector 12: S |
| | BH | OC | |

5. names of the sectors must be given.

In this situation a typical record for an individual contract would have the following layout:

- policy number: not required in this case;
- three numerical zones, the first one containing the value 1(N) or 2(S); the second one containing the values 1(SH), 2(MH) or 3(BH); the third one containing the values 1(IC) or 2(OC);
- the number of observation periods available ($\leq T$);
- for observation period i (for $i = -1, -2, \dots, -T$):
 - 1 (observation available) or 0 (observation not available);
 - numerator, the claim size for period i ;
 - denominator, the capital insured for time period i ;
 - weight, e.g. the premium paid in period i .

Because the estimate for the risk premium has to be expressed in terms of a permillage of the capital insured, the scaling factor (SF) has to be put equal to 0,001. A total of 75 bytes are necessary in this case for each record. An example of a record looks as follows:

010302 20000000 000000 0000 1000100 300000 0600 1000000 280000 0560 0000000 000000 0000

From this record we see that this contract belongs to the North (N), it is a big house (BH) outside the city (OC). There are data for two years:

- year – 2: claim size of 000100, capital insured equal to 300.000 and premium of 0600;
- year – 3: claim size of 000000, capital insured equal to 280000 and premium of 0560.

In the second part a franchise deductible policy is discussed. The question to be answered in this article is the following: which discount on the risk premium can be given by introducing a deductible. A prac-

tical example will be given to illustrate the possibilities of credibility.

In the third article we discuss a problem of health insurance. Starting from a real portfolio of contracts a risk premium will be calculated on three different levels: the overall portfolio level, the in – between or sector level (where sectors have to be chosen and defined by the user) and the individual policy – level. Examples of the output of the program will be given and discussed. This will give more insight and understanding of the previously discussed theoretical aspects and will point the way to the practical possibilities of credibility models.

References

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