Complexity and chaos in economic cybernetic systems. Chaotic models of economic systems functionality (Profit Non-linear Feedback Mechanism)

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Abstract: *In over 50 years from the cybernetics appearance, the view on cybernetic systems has a continue growth. The specialists talk about three big stages in this evolution: the* ***first class cybernetics****, in which the cybernetic system haven't a mathematical formalization;* ***the second class cybernetics*** *in which different formal models of the cybernetic systems appear and, finally,* ***the third class cybernetics*** *which assimilates and extends different new theories strong related with the general theory of the cybernetic systems.*
The mainly characteristic of this new direction is the assimilation among cybernetics methods of the new theories such as ***the dynamics of the non-linear systems, chaos theory, complexity theory, synergetic****, and others. Even if, in appearance, it's independent, those new approaches of the real systems appeals in facts many methods and theories that are components of cybernetics. In this situations, the attempt to assimilates and integrates its among the others cybernetics conceptions it's natural.*
Key words: *Cybernetic systems; Dynamic systems; Chaos; Chaotic systems; Determinist chaos; Stochastic chaos; Attractor fixed point; Chaotic trajectory.*

1 From the determinist to the chaotic systems

Until recently, cybernetics studies the systems as determinists ones, meanly as systems that could be describes in equational form, in which case the solutions obtained represents with a great deal of certainty the real system acts. But often that it isn't the true: the real system's behaviour is influenced by to many factors for could be completed described by the mathematical instruments. The great complexity of the systems has important influences on the behaviour of the cybernetic systems.
But not only that: it was observed that even for very simple systems the behaviour could be very complicated. This observation has conducted to the concepts of *chaotic system* and *chaotic systems theory (CST)*.

2. Chaos theory' origins. Principles of CST

Today, Edward Lorenz, an American weather forecaster, is considered as the CST's "father". In 1960, he makes an experiment in which he simulates different scenarios on weather evolution using a twelve-equation system solve on the computer. After a period of time he review a sequence of calculus and, for time saving, he start the experiment from the middle of the sequence by introducing in the computer the number 0.506 instead of 0.506127 used in his first experiment. After one hour, he realise that his new result are completely different from the first, even if the changes could be appreciate as insignificant at that time.
Because he use a number very closed to the first, he expect that the differences in the results being directly related with the original error, namely a small ones. But his expectations seem to be false. In this experiment Lorentz has discovered one of the fundamental properties of the chaotic systems, ***the sensitive dependence on the initial conditions***: even an imperceptible change in those conditions could generates drastic change in the long term behaviour of a system.

Starting from here, Lorentz realised that it's impossible to make an exact forecast on the weather and, in the same time his discover conduct at the chaos theory appearance. In this way he make a new step on the road opened by the great French mathematician Henry Poincaré which, in 1892, in his essay "Science and Method" has enounced the above mentioned principle for the first time, even if he doesn't realise it's huge importance for the science.

Lorentz great merit consists in his attempts to discover other systems from nature for which this propriety works. Using his twelve equations weather model, he has created a three-dimensional differential system named 'Lorentz' equations':

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = r \cdot x - y - xz \\ \dot{z} = xy - bz \end{cases}.$$

For certain values of the ***a***, ***r***, and ***b*** parameters, the solution of this system generates a chaotic trajectory in the three-dimensional space called 'Lorentz's butterfly'. Specific to this trajectory is the fact that it never cross with him self, meaning that the system never repeat exactly. Starting from this observation, Lorentz makes a huge step proving that the real systems which are dynamic, complex and non-linear, never reproduces them past evolutions. Because of that, the chaotic systems appear as untidy, even random. But they aren't like that. The random behaviour implies a certain order, and the real random systems aren't chaotic, because its could be represented using an appropriate probability distribution.

For this reason chaos couldn't be represented as an random move. It represents a third type of behaviour after the determinist and the random ones. The chaotic movement appears both in the simpler deterministic systems ('determinist chaos'), and into the extremely complex stochastic systems ('stochastic chaos').

No mater what kind of chaos we find in the natural systems, they have some common proprieties which gives the chaos theory a great generality and comprehensiveness. Even this theory it's now in the middle of its evolution, we could enounce those proprieties that characterise all dynamic, non-linear and complex system, meaning all potential chaotic systems.

a). The principle of the inherent complexity

Even the simpler dynamic determinist system could reach an extremely complicated behaviour when one of it's included process its repeated again and again until it becomes unstable therefore chaotic.

b). Lorentz's principle ('dandelion effect')

Little perturbations in the initial conditions of a system could generates major behavioural changes on the long run, fact that makes impossible the long run prediction over the system evolution.

c). The 'order in disorder' principle

On the short term the system's behaviour it's predictable because there is a gap between the moment of a little change and it's effect apparition. This gap could be used in order to make predictions over the system behaviour.

d). The inherent instability principle

Chaotic systems are functions far from equilibrium into a state of instability. A new equilibrium state appears in the system only if the instability was entirely traversed.

e). The spontaneous self-organisation principle

The pass to a new order state will be made through a series of choices in critical points, which are not predictable. In those points, the system' components co-operates, reaches a consent and organise them self into a new structure. This new structure it's dissipating, being in fact the beginning of a new transition to the chaos.

All those principles are valid for any natural systems, economic systems included. The most developed application of the chaos theory in the economic field is represented by the *Capital Market study*. It's admitted the fact that the Capital

Market, the stock exchange especially, represents a non-linear system, and chaos theory study precisely those kind of systems. Close by the models for the Capital Market, recently it was elaborates a series of models which describes economic cycles and fluctuations, economic growth process, inflation, economic politics effects. We will try bellow to illustrate those principles with a non-linear chaotic model of profit.

3. Chaotic Models of Economic Systems Functionality

(Profit Non-linear Feedback Mechanism)
Each component of the cybernetic firm's model includes one or several feedback mechanisms for control and regulation of the material and informational flows between sub-systems or between those and external environment. We will show how such a mechanism (i.e. non-linear profit' mechanism - *Stacey, 1992*), its incorporate into the sub-system profitability-costs. We all ready know that the profit and the profitability are the mainly criteria of performance for every company. As usual, the profit gained last year will have a very important role over the decision regarding the structure of the expenses for this year. In fact, these structure its dependent of the decision of profit' distribution to the finish of the last year.

In the follows we will consider that the expenses made by the firm in current period will generate a certain profit at the end of the period. From a profound analysis of this relation, we observe that the profit at the end of current period, P_t, is influenced by the structure of the expenses in this period, and it influence the profit in the past period, P_{t-1}, because the desire for a large profit this year determines the adoption of a certain decision regarding the distribution of the profit obtained in the previous year.

The figure no. 1 shows the feedback relation between the two levels of the profit, from the current and from the previous period.

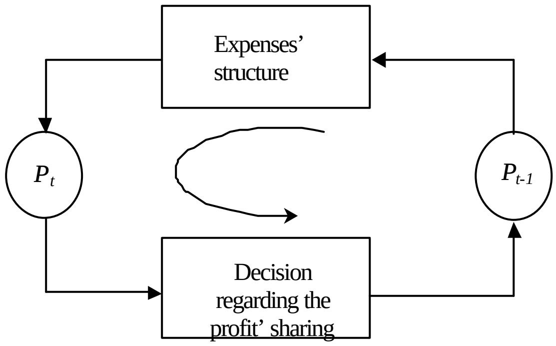

Figure 1

The dependence between actual profit, P_t and the previous, P_{t-1} is not a linear one because the expenses' growth generates the future profit growth only to a certain limit, beyond which rinsing the expenses will determine the reducing of the actual profit (see the advertising expenses example). In order to illustrate this thing we will use a non-linear relation as:

$$P_1 = AP_0 - BP_0^2. \quad (1)$$

If B<<A then, for small values of P_0, the second term from the right side of the relation its insignificant. The minus shows that the second term tends to decrease the profit. Using the relation (1), we could writes for the next periods:

$$P_2 = AP_1 - BP_1^2$$

$$P_3 = AP_2 - BP_2^2$$

…………………

$$P_{t+1} = AP_t - BP_{0t}^2.$$

Suppose that there is a maximum possible profit' level, P^{max}. We observe then, in order that P_{t+1} >0, is necessary that P^{max}<A/B. Indeed, from

$$P_{t+1} = AP_t - BP_t^2,$$

and, with P_t=A/B we obtain:

$$P_{t+1} = A\cdot\frac{A}{B} - B\cdot\frac{A^2}{B^2} = \frac{A^2}{B} - \frac{A^2}{B} = 0 \quad (2)$$

If P_t <A/B then P_{t+1}>0, and if P_t >A/B then P_t<0.

In this case we could introduce a new variable:

$$p_t = \frac{P_t}{P^{max}},$$

which express the profit obtained by the firm in year *t* as a fraction from the maximum possible profit. Obvious that $0<\pi_t<1$.

The previous relations divided at P^{max} leads to:

$$\frac{P_{t+1}}{P^{max}} = A\frac{P_t}{P^{max}} - B\left(\frac{P_t}{P^{max}}\right)^2 P^{max},$$

or, using the new variable π_t:

$$p_{t+1} = Ap_t - Bp_t P^{max}.$$

But, taking $P^{max} = \frac{A}{B}$:

$$BP_t = B\frac{P_t}{P^{max}}P^{max} = Bp_t\frac{A}{B} = Ap_t.$$

Finally, we have:

$$p_1 = Ap_t - Ap_t^2 = A(1-p_t)p_t.$$

The relation

$$p_{t+1} = Ap_t(1-p_t) = f_A(p_t),$$

it's called also the ***logistic application***, one of the basic relation analysed by the theory of the chaotic systems.

Let see, as follow, what this relation tells us about the profit long term behaviour, and how *p* depend on constant *A*. The linear intuition make us to expect that, if the parameter *A* has is constant, *p* should have well defined values. More then that, we could expect that the profit will change gradual if a will change gradual.

The calculus could be made as follow:

We start with a certain given value π_0; then we calculates π_1, π_2, ... and so on.

$$p_1 = f_A(p_0); \quad p_2 = f_A(p_1); \quad p_3 = f_A(p_2)...$$

In this way we obtain a sequence of iterations, which generates a sequence of values for the profit π called *trajectory* or *orbit*.

Of course, the first values of this trajectory depend on initial value of π. What isn't obvious in this moment is the fact that the trajectory' possible behaviour its the same for all most all the initial values π_0 between 0 and 1 for a given value of *A*.

Yet, same initial values are different from the others. Thus, if we get $p_0=0$ we observe immediately that $f_A(p_0)=0$ and the trajectory is constant at $\pi=0$ for all the next iterations.

A value of π, noted π^*, for which we have

$p^*= f_A(p^*)$,

is called *fixed point* of the iterative application f_A. For the logistic application there are two fixed points which are obtained by solving the equation:

$p^*=Ap^*(1-p^*)$.

Its observed that a solution is $\pi_1^*=0$, and the other is $\pi_2^*=1-1/A$. For A<1, $\pi_1^*=0$ its the only fixed point, which interested for the application, in this case π_2^* being outside the interval [0,1]. If A>1 both fixed points are included into the domain [0,1], so its are interested.

In order to see the importance of those fixed points we give the graphic representation of the function $y= f_A(p)$, for many values of A, with 0<A<4. For A<1, as we know, we have a single fixed point equal to zero (see figure no. 2) which correspond to the cross of $f_A(p)$ with the diagonal $y=p$.

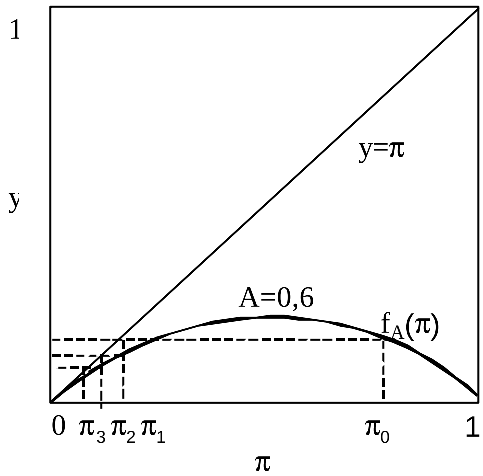

Figure 2

The evolution of the profit in this case is obtained as follow:

We consider an initial value of the profit, π_0, on the π axis, and we draw a vertical line to the f_A curve. Then, we project the intersection point on the *y*-axis. The intersection of this projection with the main diagonal y=π determines the co-ordinates for the next point π_1 on the π-axis. Again, the vertical line from π_1 intersects the f_A curve into a point which' projection on the y-axis determines, through it's intersection with the main diagonal y=π, the co-ordinates for the next point π_2, and so on.

In this case, we observe that the set of values π_0, π_1, π_2,.., obtained for A<1 tends to 0, that is the fixed point of the application $f_A(p)$. For this reason, $\pi^*=0$ is also *called attractor fixed point*, or just attractor for the iterative application $y=f_A(p)$. The interval $0\le\pi\le1$ is called *attraction field* for the respectively attractor, because each trajectory which have a point included into the interval [0,1] as initial value, will tends to $\pi^*=0$ with each iteration.

In the terms of the considered economic example, we concludes that, if A<1, the firm will registered a smaller profit ($\pi_t\to0$) when *t* rise.

Let consider now the case in which 1<A<3. For example, if we take A=1,5 we obtain two fixed points $\pi_1^*=0$ and $\pi_2^*=1/3$. By choosing the initial condition $\pi_0=0{,}1$ and proceeding as above, we observe that the trajectory tends to the fixed point $\pi_2^*=1/3$ (figure 3).

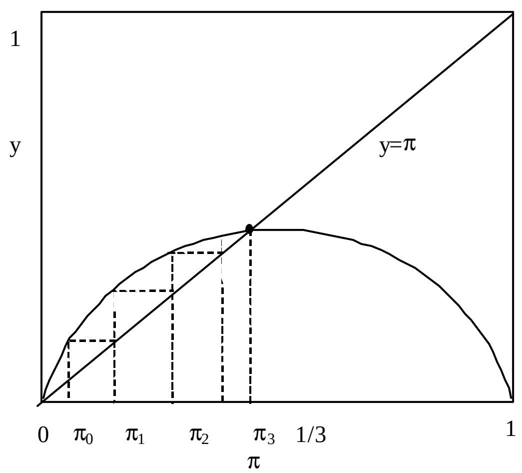

Figure 3

By changing the initial condition for any other $\pi_0\in(0,1)$, the obtained trajectory tends to $\pi=1/3$. So this fixed point is attractor for any value π from the attraction field (0,1). We will obtain the same thing for any function $f_A(\pi)$ for which A takes values between 1 and 3.

Instead, if $3\le A\le4$, even if no other fixed points appear the trajectory sift to something else but a fixed point. Thus, in figure 4 is represented the profit' trajectory in case A=3, in figure 5 is represented the profit' trajectory in case A=3,5, and in figure 6 is represented the profit' trajectory in case A = 3,54.

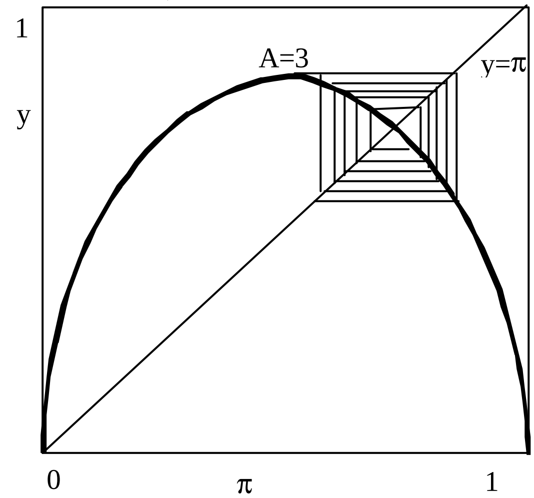

Figure 4

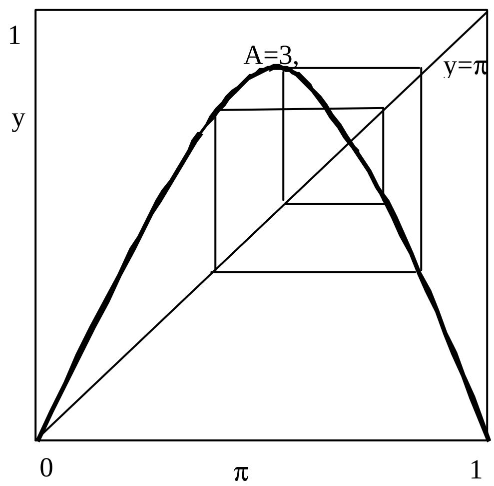

Figure 5

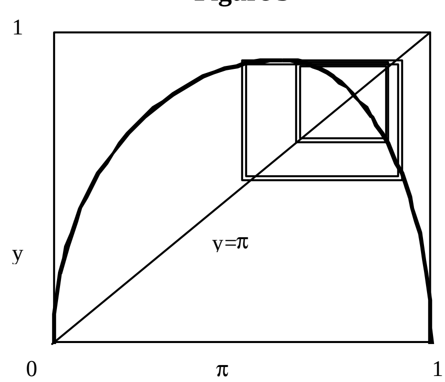

Figure 6

We observe that, when A rise the trajectory becomes more complicated, sifting from the 'cobweb' form to the periodical alternation between two values. In this

final case we obtained a ***two periods cycle*** (figure 6).
If A continue to change, we will obtain double periods sequence, so we obtain cycles with four periods, eight periods and so on.
Finally, for A≥3,57 we obtain a chaotic trajectory (figure 7).

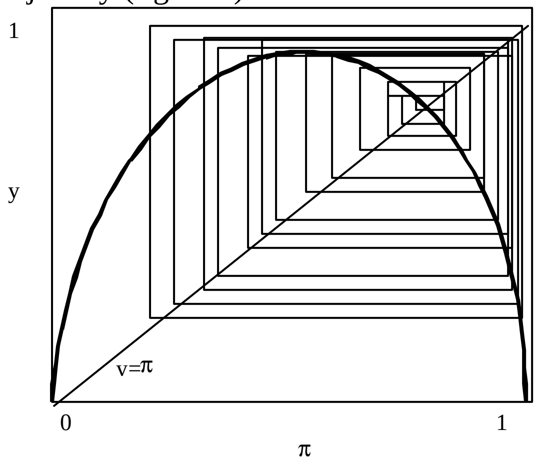

Figure 7

We observe now that, no mater the initial condition is, after *t* iterations the profit could rich any point into the interval [0,1]. Also, any modification in the initial condition, even a smaller one, will offer a trajectory total different from the initial one. In this case we say that the dynamics of the non-linear feedback profit mechanism its a chaotic one when 3,57<A<4. For A taking values bigger than 4, the dynamic become again a deterministic one, so it is for the interval [1, 3].
The above example shows that the switching between the deterministic and the chaotic behaviour could be obtained even for smaller changing of a parameter (in our case A), wich depend on a lot of internal and external factors that influences the firm activity. Because this switch is possible anytime, the analysis of the condition of appearance of the chaotic behaviour in the economic systems it's totally justified.
The non-linear dynamic economic systems approach through chaotic systems' theory is very well developed today. Thus they are created the chaotic versions for one of the classic deterministic models such as Samuelson, Hicks, Kaldor s.a. Also it was created original models for economic systems which incorporate the chaos, and which reproduces a very wide spectrum of possible behaviours into an economic medium extremely volatile.

References

1. Baker, G., Gollub, J., **"Chaotic Dynamics. An Introduction"**, Cambridge University Press, New York, 1990;
2. Froyland, J., **"Introduction to Chaos and Coherence"**, C. P., Oxford, 1990;
3. Gleick, J., **"Chaos, Making a New Science"**, Viking, New York, 1987;
4. Hall, N., **"The New Scientist Guide to Chaos"**, Penguin Books, 1991;
5. Hilborn, R. C., **"Chaos and Non-linear Dynamics: Introduction for Scientist and Engineers"**, Oxford University Press, New York, 1994;
6. Mandelbrot, B., **"Obiectele fractale: form◈, hazard, dimensiune"**, Nemira Publishing, Bucharest, 1998;
7. Mullin, T., **"The Nature of Chaos"**, Clarendon Press, Oxford, 1993;
8. Ruelle, D., **"Chance and Chaos"**, Penguin Books, 1993;
9. Sarben, S., **"Non-linear Phenomena and Chaos"**, Adam Hilger Ltd., Bristol, 1986;
10. Wiggins, S., **"Global Bifurcations and Chaos – Analytical Methods"**, Springer-Verlag, New York, 1988;
11. Zhang, W. B., **"Synergetic Economics. Time and Change in Non-linear Economics"**, Springer-Verlag, Berlin, 1991.